
Manual of Weighing Applications

Part 1

Density



Foreword

In many common areas of application, a weighing instrument or the weight it measures is just a means to an end: The value that is actually sought is calculated from the weight or mass that was determined by the weighing instrument. That is why this **Manual of Weighing Applications** describes the most widely used weighing applications in a series of separate booklets, with each booklet representing a complete and independent manual.

Each of these booklets begins with a description of the general and theoretical basis of the application concerned – which in many cases cannot be done without using formulas and equations taken from the fields of physics and mathematics. The equations used are explained in the text, and the intermediate steps necessary for arriving at the results given here are also included. Some readers may get the impression, at first glance, that the manual is just a collection of (many) equations, but we do hope that the most important points are familiar to all readers, even in this context.

Each separate booklet also contains a chapter giving detailed examples of applications, as well as an index where you can look up keywords to find the information you need.

At the back of each booklet, just before the index, we have included a list of questions on the subject so that readers can check whether they have understood what they have read and can put it into practice.

This manual was written to provide **sartorius** employees and associates, as well as any interested customers, with a comprehensive compilation of information on the most widely used applications, both to introduce the subject and to increase existing knowledge in the field, as well as to provide a reference source. We also hope that readers who make active use of this manual will gain confidence in finding solutions when using these applications and gain a feeling for the possibilities that these weighing applications open up, so they can begin to create custom solutions where needed.

Contributions from users both in the laboratory and in industry will help ensure that this manual "lives" and grows with use. We are especially interested in receiving your application reports, which we would like to include in future editions of the **Manual of Weighing Applications**, to make information about new and interesting applications of weighing technology available to all our readers.

Marketing, Weighing Technology
February 1999

Symbols

Indices

a	Air
fl	Fluid (flowing system)
s	Solid
(a)	Determined in air
(fl)	Determined in a fluid
B	Buoyancy
b	Bulk
t	Total

Symbol	Unit	Symbol	Unit
m	Mass	m	kg
V	Volume	V	m ³
A	Area	A	m ²
F	Force	F	N = kg·m/s ²
W = m·g	Weight (force)	W = m·g	N = kg·m/s ²
g	Gravitational acceleration	g	m/s ²
p = F/A	Pressure	p = F/A	Pa=N/ m ²
ρ rho	Density	ρ rho	kg/ m ³ ; g/cm ³
T	Temperature in Kelvin	T	K
t	Temperature in °Celsius	t	°C t=T-273,15 K
γ gamma	Specific gravity (old!)	γ gamma	kp/dm ³
α alpha	Linear expansion coefficient of (of solids)	α alpha	1/K = K ⁻¹
γ gamma	Expansion coefficient (of liquids or gases)	γ gamma	1/K = K ⁻¹
φ phi	Relative humidity	φ phi	%
π pi	Porosity	π pi	Volume-%

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The General Principles of Density Determination

Examples of Applications for Density Determination

Density is used in many areas of application to designate certain properties of materials or products. In conjunction with other information, the density of a material can provide some indication of possible causes for alterations in product characteristics. Density determination is among the most often used gravimetric procedures in laboratories.

Density can indicate a change in the **composition of a material**, or a defect in a product, such as a crack or a bubble in cast parts (known as voids), for instance in sanitary ceramics or in foundries in the iron and steel industries.

In aluminum foundries, the **melt quality** is monitored by taking two samples: one under air pressure and one under, for example, 80 mbar pressure. Once they have set and cooled, the density of the samples is determined. The ratio of both density values provides information on the purity of the melt.

Determining the density of plastics used in engineering can help to monitor the **proportion of crystalline phase**, because the higher geometric order of crystals makes them denser than the non-crystalline portion. The density of glass is determined by both the chemical composition of the sample and the cooling rate of the melted mass.

With **porous materials**, the density is affected by the quantity of pores, which also determines certain other qualities of the material; for instance, the frost resistance of roof tiles, or the insulating properties of wall materials such as clay and lime malm bricks or porous concrete.

One of the factors measured to determine the quality of wine is known as the must weight, which is measured in degrees Öchsle – this is also a density value, because the **density is proportional to the concentration of a substance** in the solvent (e.g., sugar, salt or alcohol in an aqueous solution).

The density of products also plays an important role in the **average weight control of prepackaged products**, in those cases where a package is **filled by weight** but must carry a label **indicating the contents in volume**.

Density: A Definition

Density (ρ) is the proportion of mass (m) to volume in an amount of material. The terms used here are defined in the German Industrial Standard (DIN) 1306.

$$\rho = \frac{m}{V}$$

Equation 1

Different fields of industry and technology have also developed various special terms relating to density:

- **Normal density** – the density of gases under normal physical conditions (0°C and 1013 hPa)
- **Tap density** – the density of a powdery material under undefined conditions; for example, in shipping (DIN 30 900) or the mass quotient of the uncompressed dry granules in a designated measuring container divided by the volume of the container (DIN EN 1097-3)

- **Apparent density** – the density of a powder when filled in accordance with the relevant designated testing procedure; this is important for determining the fill quantity of pressing molds
- **Bulk density** – the mass/volume ratio that includes the cavities in a porous material
- **Solid density** – the mass/volume ratio of a porous material; i.e., excluding the pore volume
- **True density** – Term still used for "solid density"
- **Relative density** – the proportion of a density value ρ to a reference density ρ_0 taken from a reference material; relative density is a ratio with the dimension 1

The value for **specific gravity** is still sometimes given as well, although it is rarely used today.

$$g = \frac{W}{V} = \frac{m \cdot g}{V} \quad \text{Equation 2}$$

In contrast to the density, this value indicates the weight force in relation to the volume; i.e., the difference between density and specific gravity is that the calculation of specific gravity includes the gravitational acceleration (g).

Units for Measuring density

In the International System of Units, the unit for density is kg/m^3 ; the unit used most often is g/cm^3 – this corresponds to the results in g/ml . Results are also sometimes given in kg/dm^3 .

$$1 \text{ kg/m}^3 = 0.001 \text{ g/cm}^3 \quad \text{or:}$$

$$1 \text{ g/cm}^3 = 1 \text{ kg/dm}^3 = 1000 \text{ kg/m}^3$$

Dependence of Density on Temperature

The density of all solid, liquid and gaseous materials depends on **temperature**.

Aside from temperature, the density of gaseous materials also depends on pressure. Gases are compressible at "normal" pressure; this means that air density changes when the air pressure changes.

The **normal density** is the density of a gas (or combination of gases) under normal physical conditions: temperature $T = 0 \text{ }^\circ\text{C}$, pressure $p = 101.325 \text{ kPa}$.

A general rule is: **The higher the temperature, the lower the density.** Materials expand when heated; in other words: their volume increases. Therefore the density of materials will decrease as their volume increases. This is more noticeable in liquids than in solids, and especially in gases.

The change in density over a certain temperature interval can be calculated using the heat expansion coefficient; this will yield the change in volume of a material in relation to the temperature (see Appendix, page xx).

The following diagrams show the density of various substances calculated in relation to the temperature – the x axes show density in intervals of 0.06 g/cm^3 (except in the case of air).

As can be seen from these diagrams, temperature affects some substances more strongly than others. **For density determination, this means that – depending on the required accuracy of measurement, of course – the test temperature must be set very precisely and kept constant.**

In hydrostatic density determination methods, for example, it is usually better to use water than ethanol as a liquid for buoyancy; when the temperature increases, for example, from 20°C to

21°C, the density of the water only decreases by 0.00021 g/cm³, where the density of ethanol decreases by 0.00085 g/cm³ – more than 4 times as much. This means that the temperature has to be controlled more precisely, or a greater error must be assumed in the results of the density determination using ethanol.

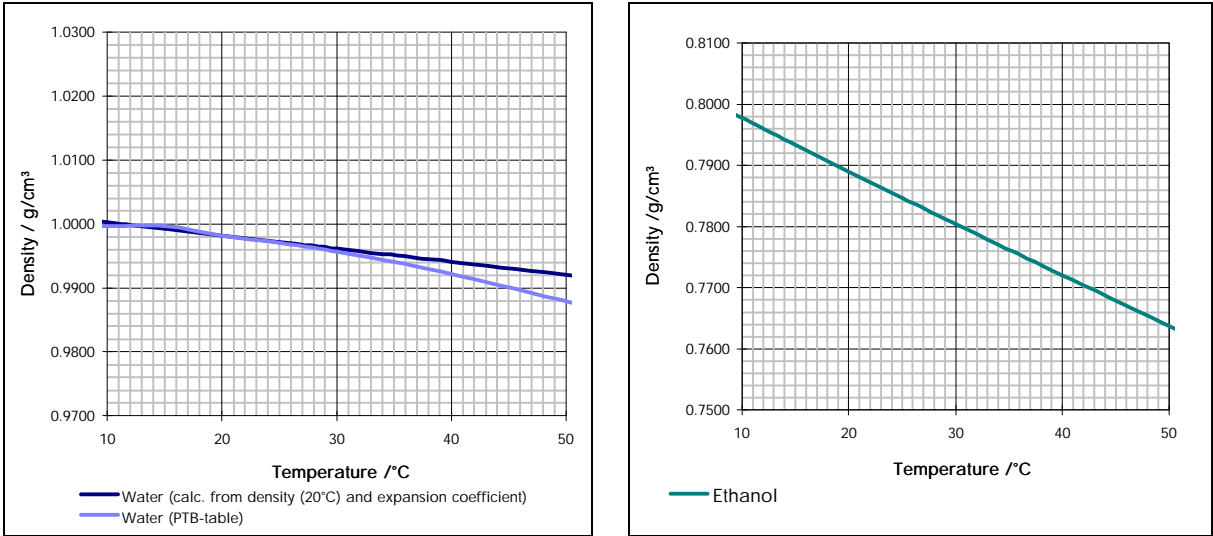
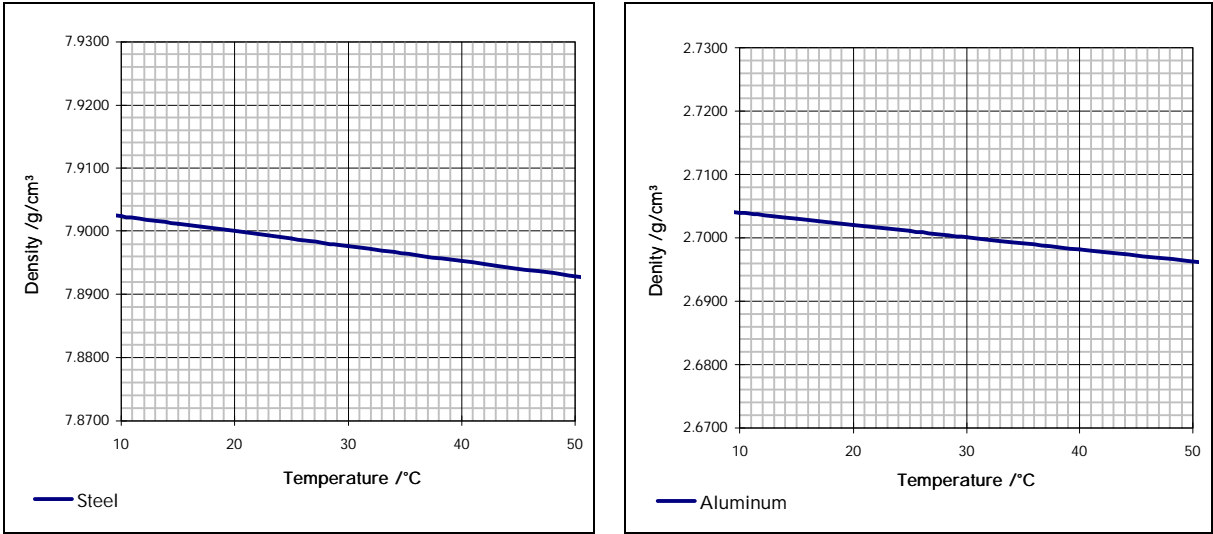


Figure 1: Temperature dependency of density for water and ethanol (above) and for steel and



aluminum (below)

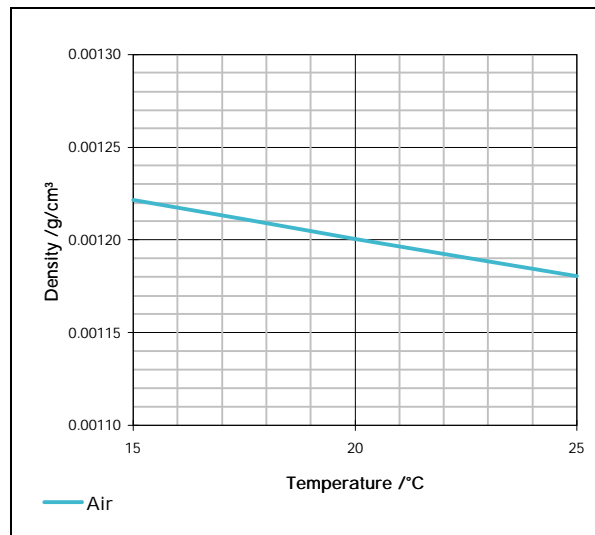
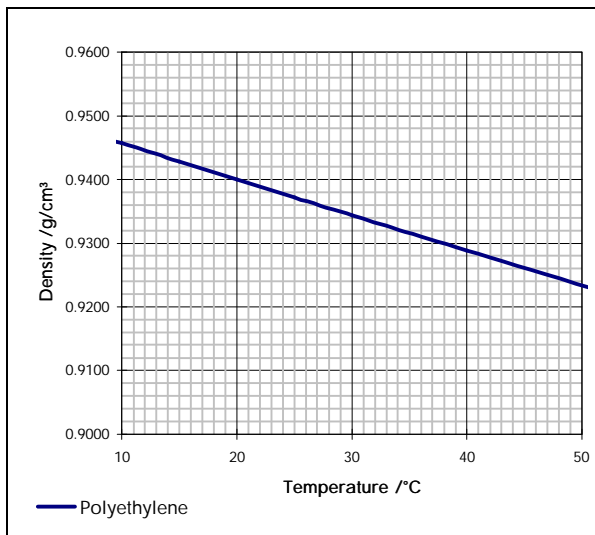
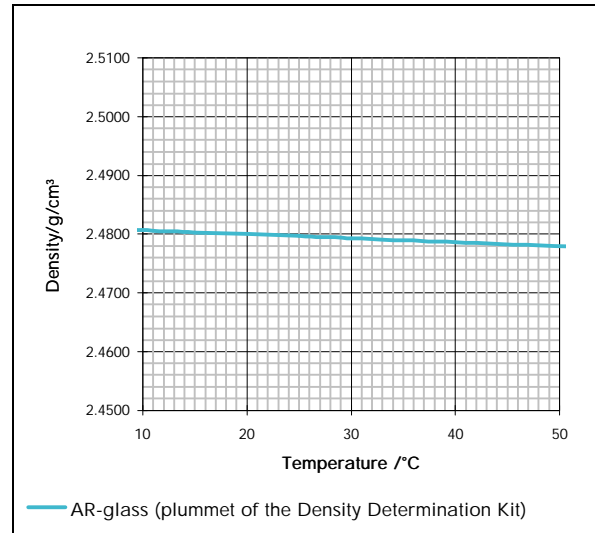
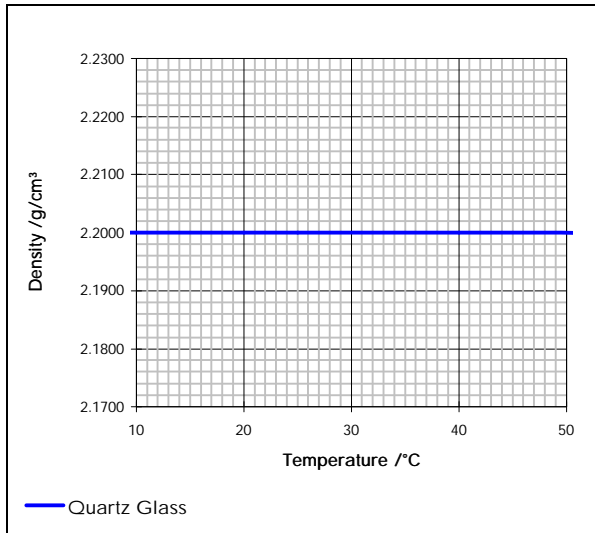


Figure 2: Temperature dependency of glass, polyethylene and air

The Archimedean Principle

In accordance with the definition of density as $\rho = m/V$, in order to determine the density of matter, the mass and the volume of the sample must be known.

The determination of mass can be performed directly using a weighing instrument.

The determination of volume generally cannot be performed directly. Exceptions to this rule include

- cases where the accuracy is not required to be very high, and
- measurements performed on geometric bodies, such as cubes, cuboids or cylinders, the volume of which can easily be determined from dimensions such as length, height and diameter.
- The volume of a liquid can be measured in a graduated cylinder or in a pipette; the volume of solids can be determined by immersing the sample in a cylinder filled with water and then measuring the rise in the water level.

Because of the **difficulty of determining volume with precision**, especially when the sample has a highly irregular shape, a "detour" is often taken when determining the density, by making use of the **Archimedean Principle**, which describes the relation between forces (or masses), volumes and densities of solid samples immersed in liquid:

From everyday experience, everyone is familiar with the effect that an object or body appears to be lighter than in air – just like your own body in a swimming pool.

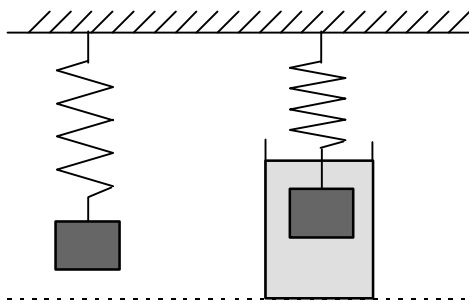


Figure 3: The force exerted by a body on a spring scale in air (left) and in water (right)

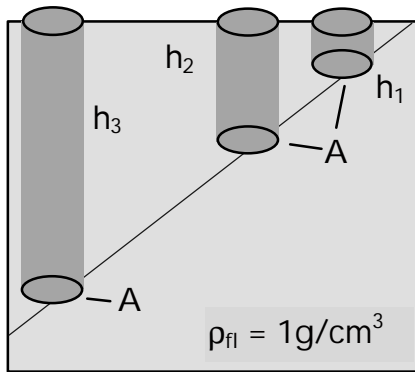
Both the cause of this phenomenon and the correlation between the values determined in its measurement are explained in detail in the following.

A body immersed in water is subjected to stress from all sides simultaneously due to hydrostatic pressure. The **horizontal stress** is in equilibrium, which is to say that the forces cancel each other out.

The **vertical pressure** on the immersed body increases as the depth of the body under the surface increases. The pressure at a certain depth in liquid exerted by the liquid above that point is called **weight pressure**. The weight pressure can be calculated from the density of the liquid, the height of the liquid and the gravitational acceleration: $p = \rho_{fl} \cdot g \cdot h$.

The same pressure is exerted on area A at depth h :

$$F = p \cdot A = \rho_{fl} \cdot g \cdot A \cdot h \quad \text{Equation 3}$$



with $\rho_{fl} = 1 \text{ g/cm}^3$ and $g \cong 10 \text{ m/s}^2$				
	h	p	A	F
1	1 cm	0.01 N/cm ²	1 cm ²	0.01 N
2	10 cm	0.1 N/cm ²	1 cm ²	0.1 N
3	50 cm	0.5 N/cm ²	1 cm ²	0.5 N

Figure 4: Gradient of pressure in liquid

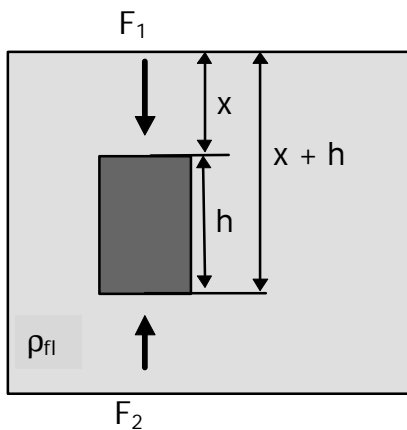


Figure 5: Buoyancy effects

The weight pressure on the surface of an immersed body with area A causes a force of $F_1 = A \cdot x \cdot \rho_{fl} \cdot g$ to be exerted on the upper surface of the body, and $F_2 = A \cdot (x+h) \cdot \rho_{fl} \cdot g$ on the lower surface. The resulting force on the body can be calculated from the difference between these two forces:

$$\begin{aligned}
 F_{res} &= F_2 - F_1 \\
 &= [A \cdot (x+h) \cdot \rho_{fl} \cdot g] - [A \cdot x \cdot \rho_{fl} \cdot g] \\
 &= A \cdot (x+h-x) \cdot \rho_{fl} \cdot g \\
 &= A \cdot h \cdot \rho_{fl} \cdot g
 \end{aligned}
 \tag{Equation 4}$$

The product of area and height of the body is equal to the volume of the body. This volume is in turn equal to the volume of water that is displaced by the immersed body.

$$(A \cdot h) = V_s = V_{fl}
 \tag{Equation 5}$$

Thus the resulting force is

$$\boxed{F_{res} = V_{fl} \cdot \rho_{fl} \cdot g = F_B}
 \tag{Equation 6}$$

This force is known as **buoyancy force** or simply **buoyancy**, and it directly includes the value for volume to be determined.

Observing the ratio of forces exerted on the immersed body and the water displaced by the body, it can be seen that the forces exerted include both the weight W_s , a downward force, and buoyancy F_B , an upward force. The resulting force can be calculated from the difference between these two forces: $F_{res} = W_s - F_B$. The buoyancy F_B exerted on the body is equal to the weight W_{fl} of the liquid displaced by the body.

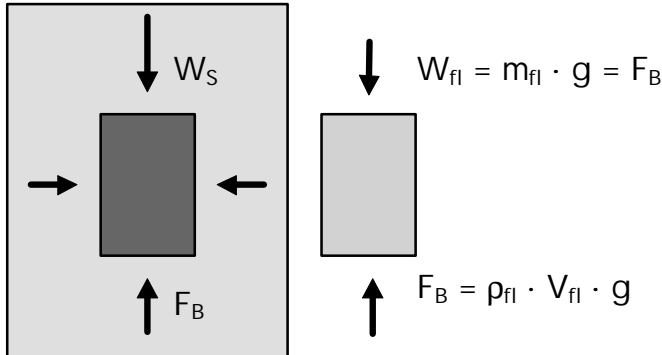


Figure 6: On the Archimedean Principle: The figure on the left represents the body immersed in liquid; on the right, the liquid element.

The result is

$$W_{fl} = m_{fl} \cdot g = \rho_{fl} \cdot V_{fl} \cdot g \quad \text{Equation 7}$$

If the body and the liquid element are at equilibrium, the buoyancy F_B must, by module, be equal to the weight W_{fl} ; thus

$$F_B = W_{fl} \quad \text{Equation 8}$$

The buoyancy is the result of the level of hydrostatic pressure in a liquid. Buoyancy is inverse to the weight of a body immersed in liquid. This explains why a body seems to be lighter in water than in air. Depending on the ratio of body weight to buoyancy, the immersed body may sink, float or be suspended.

If the buoyancy is less than the weight ($F_B < W_s$), **the body will sink**. In this case, the density of the body is greater than that of the liquid ($\rho_s > \rho_{fl}$). The widely used method of determining density according to the buoyancy method is usually used under these conditions.

If the buoyancy is equal to the weight ($F_B = W_s$), **the body remains completely immersed and is suspended in the liquid**. Because both the volume and mass of the body are equal to the volume and mass of the displaced water, it follows that the body and the liquid have the same density. There are a number of density determination procedures that make use of this condition (see Page 18).

If the buoyancy is greater than the weight ($F_B > W_s$), **the body floats**; i.e., it rises to the surface of the liquid and remains only partially immersed. In fact, it dips so far into the liquid until the weight of that volume of liquid that is displaced is equal to the weight of the body. In this case, the volume of the displaced liquid is less than the volume of the body ($V_{fl} < V_s$) and the density of the liquid is greater than the density of the body $\rho_{fl} > \rho_s$. These are the conditions for density determination using a hydrometer (see page 20).

Gravimetric Methods of Density Determination

Density Determination Based on the Archimedean Principle

The relationships between the mass, the volume and the density of solid bodies immersed in liquid as described by Archimedes form a basis for the determination of the density of substances. The difficulty in this method of density determination lies in the precise determination of the volume of the sample.

When a body is completely immersed in liquid, the mode of procedure demands that the volume of the body is equal to the volume of the displaced liquid. Thus we can derive the following general equation between the density and mass of a liquid and of a solid, in which the volumes are not explicitly named (see Appendix, page 54 for the derivation):

$$\rho_s = \rho_{fl} \cdot \frac{m_s}{m_{fl}} \quad \text{Equation 9}$$

Accordingly, the unknown density of a solid substance can be determined from the known density of the liquid for buoyancy and two mass values:

$$\rho_{fl} = \rho_s \cdot \frac{m_{fl}}{m_s} \quad \text{or} \quad \rho_{fl} = \frac{m_{fl}}{V_s} \quad \text{Equation 10}$$

Reciprocally, the density of liquids can be determined from one mass value and the known density of the immersed body.

Simple and precise methods of mass determination can eliminate the need to measure volume.

Hydrostatic balances and Mohr balances are still used in some cases for measuring density; the Mohr balance, a beam balance has been widely replaced by the use of density sets in conjunction with laboratory balances.

There are two basically different procedures for hydrostatic weighing methods. The actual meaning of the values displayed on the weighing instrument depends on the mode of procedure used. The buoyancy method (see Figure 7 and Figure 8) entails measuring the weight of the body, which is decreased by buoyancy, while the displacement method (see Figure 9) calls for the direct measurement of the weight or mass of the displaced fluid.

Other methods that are based on the Archimedean principle include density determination using hydrometers (see page 20) as well as various suspension methods (see page 18).

Buoyancy Method

The buoyancy method is often used to determine the density of bodies and liquids. The apparent weight of a body in a liquid, i.e., the weight as reduced by the buoyancy force is measured. This value is used in combination with the weight in air to calculate the density.

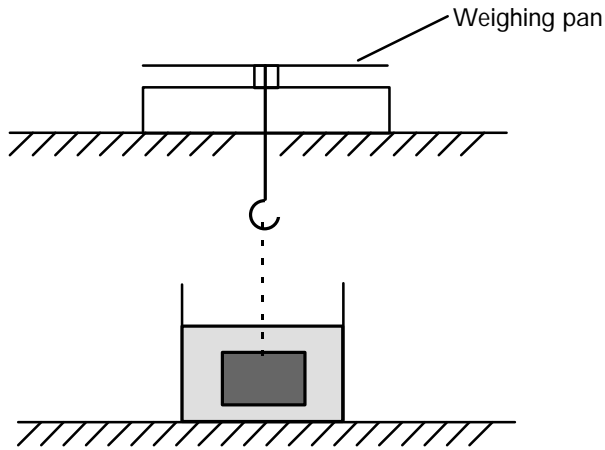


Figure 7: Basic procedure for the buoyancy method with below-balance weighing

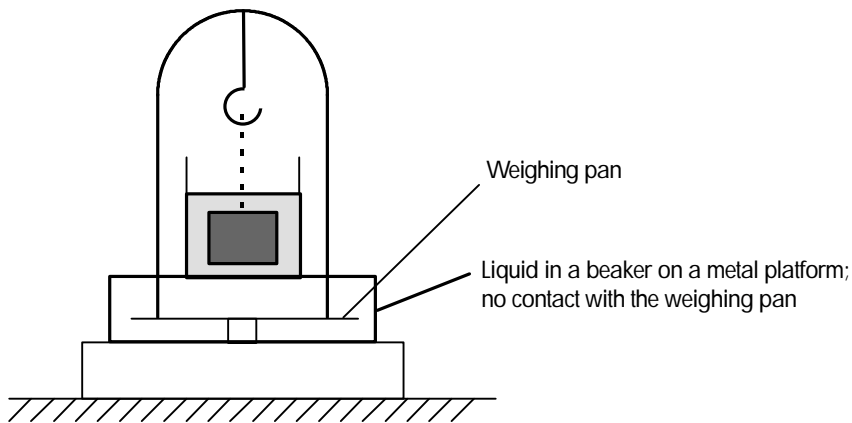


Figure 8: Basic procedure for the buoyancy method with a frame for hanging the plummet and a bridge for holding the container for liquid

In the procedures illustrated in Figure 7 and Figure 8, the values displayed on the weight readout indicate the mass of the immersed body as reduced by buoyancy (see also Figure 3).

This means that, in light of the equation $\rho_s = \rho_{fl} \cdot (m_s / m_{fl})$, the mass of the body weighed in air is known: $m_s = m_{(a)}$. The mass of the liquid m_{fl} is not directly known, but is yielded by the difference between the weights of the body in air ($m_{(a)}$) and in liquid ($m_{(fl)}$):

$$m_{fl} = m_{(a)} - m_{(fl)}$$

This changes Equation 9 for determining the density of the body into:

$$\mathbf{r_s = r_{fl} \cdot \frac{m_{(a)}}{m_{(a)} - m_{(fl)}}}$$

Equation 11

To determine the density of a liquid, m_{fl} is again calculated from the values measured for mass of the body in air and in liquid $m_{fl} = m_{(a)} - m_{(fl)}$ and the result used in Equation 10. The buoyancy method maintains the relationship for determination of the density of the liquid:

$$\mathbf{r_{fl} = r_s \cdot \frac{m_{(a)} - m_{(fl)}}{m_{(a)}} = \frac{m_{(a)} - m_{(fl)}}{V_s}}$$

Equation 12

V_s is the known volume of the plummet used to determine the liquid density. Thus the density of a substance can be determined in two weighing operations.

Displacement Method

The displacement method is another way that the Archimedean Principle is used in determining the density of bodies and liquids.

The procedure for the displacement method entails determining the mass of the liquid displaced by the body. A container of liquid is placed directly on the weighing pan while the body is immersed. In most cases, the body is suspended from a hanger assembly.

When the body is immersed in the liquid, it displaces a volume of liquid V_{fl} with density ρ_{fl} and mass m_{fl} . The buoyancy force exerted on the body is $F_B = \rho_{fl} \cdot V_{fl} \cdot g = m_{fl} \cdot g$. Because the weight of the body $W_s = m_s \cdot g$ is carried by the hanger assembly and the balance is not loaded, the balance readout directly indicates the mass of the liquid m_{fl} – assuming the weight of the container was tared beforehand.

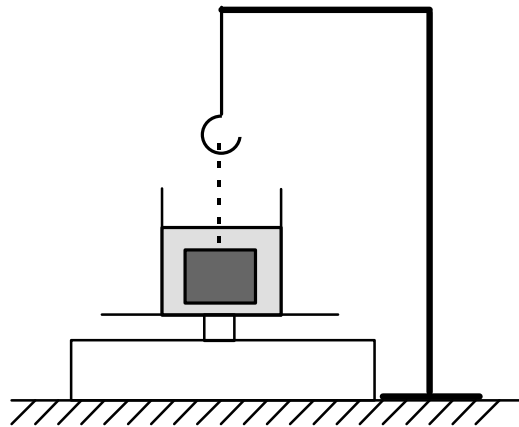


Figure 9: Basic mode of procedure for the displacement method

This means that, in the case of the displacement method, Equations 9 and 10 (see page 10) can be directly applied for density determination. For the density of a solid:

$$\mathbf{r_s = r_{fl} \cdot \frac{m_s}{m_{fl}}}$$

Equation 13

while for the density of a liquid:

$$\mathbf{r_{fl} = r_s \cdot \frac{m_{fl}}{m_s} = \frac{m_{fl}}{V_s}}$$

Equation 14

If you use a plummet with a known volume V_s , the unknown density ρ_{fl} of a liquid can be calculated from just one measurement.

Determining the Density of Air

To convert a weight value to the true mass value, you must know the value for the air density. This value can vary over the course of a day by an average of $\pm 0.05 \text{ mg/cm}^3$ in relation to the normal density of 1.2 mg/cm^3 . This is why the air density must be determined at the time the value is required, with a relative uncertainty factor of $< 5 \cdot 10^{-4}$.

The air density ρ_a depends on the temperature T , the pressure p and the relative humidity of the air ϕ . There are various approximation formulas used to determine the density of air as dependent on air pressure, temperature and humidity, and even some which consider of the CO_2 -content of the air (see also "The Fundamentals of Weighing Technology," pp.47-48.)

It is also possible to determine the density of air (with a $\approx 1\%$ margin of error) with high-resolution weighing instruments. This is done using 2 calibrated weights that are each made of different materials, with different densities (for example, aluminum and steel).

This determination method is also based on the Archimedean Principle. Because air is made up of matter, a body in air is subject to buoyancy just as it is in liquid. The same regularities apply as those described in the chapter entitled "The Archimedean Principle", on page 7.

Observing – first in a vacuum – an aluminum cylinder with a density $\rho_{\text{Al}} \approx 2.7 \text{ g/cm}^3$ (Figure 10, left), we see that it is in equilibrium with a standardized weight of the same mass ($\rho_{\text{N}} = 8.000 \text{ g/cm}^3$).

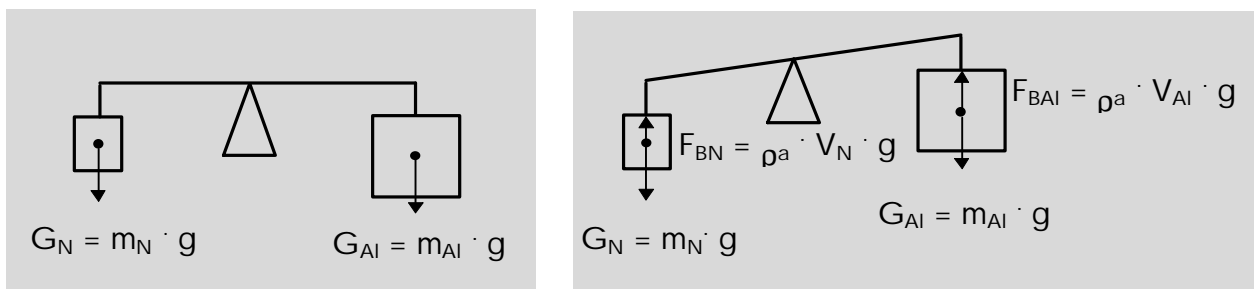


Figure 10: The effect of buoyancy on weighing in a vacuum (left) and in air (right)

Observing the same circumstances in air (Figure 10, right), rather than in a vacuum, we see that the aluminum cylinder and the standardized weight are no longer in equilibrium. This is due to the difference in buoyancy forces caused by the different material densities and volumes.

To determine what mass m_{N} holds the aluminum cylinder (m_{Al}) in equilibrium in air that has a density ρ_a , all effective forces are observed at equilibrium:

$$W_{\text{N}} - F_{\text{BN}} = W_{\text{Al}} - F_{\text{BAI}}$$

Equation 15

$$\underbrace{m_{\text{N}} \cdot g}_{\text{weight}} - \underbrace{\rho_a \cdot V_{\text{N}} \cdot g}_{\text{buoancy}} = \underbrace{m_{\text{Al}} \cdot g}_{\text{weight}} - \underbrace{\rho_a \cdot V_{\text{Al}} \cdot g}_{\text{buoancy}}$$

Conversion with $V_{\text{N}} = \frac{m_{\text{N}}}{\rho_{\text{N}}}$ and $V_{\text{Al}} = \frac{m_{\text{Al}}}{\rho_{\text{Al}}}$ then yields

$$m_{AI} = m_N \cdot \frac{1 - \frac{r_a}{r_N}}{1 - \frac{r_a}{r_{AI}}} \quad \text{Equation 16}$$

m_N is the weight value W . The weight value is in general equal to the readout on the weighing instrument. The weight value $W_{AI} = m_{AI} \cdot \frac{1 - \frac{r_a}{r_{AI}}}{1 - \frac{r_a}{r_N}}$ is not constant; rather, it is dependent on the air density – on the weather, so to speak. This relationship applies in a similar manner for the steel cylinder in the weight set for the determination of air density: $W_{St} = m_{St} \cdot \frac{1 - \frac{r_a}{r_{St}}}{1 - \frac{r_a}{r_N}}$. From these 2 equations, a relation to the determination of the air density can be derived (see Appendix, page 57):

$$r_a = \frac{m_{AI} \cdot W_{St} - m_{St} \cdot W_{AI}}{\frac{m_{AI} \cdot W_{St}}{r_{AI}} - \frac{m_{St} \cdot W_{AI}}{r_{St}}} \quad \text{Equation 17}$$

W_{St} and W_{AI} are the weight values currently measured.

m_{St} and m_{AI} are calculated according to the following formula, using the conventional mass and the densities of the certified weights:

$$m_{St} = M_{St} \cdot \frac{1 - \frac{1.2 \text{ kg/m}^3}{8000 \text{ kg/m}^3}}{1 - \frac{1.2 \text{ kg/m}^3}{r_{St}}} \quad \text{or} \quad m_{AI} = M_{AI} \cdot \frac{1 - \frac{1.2 \text{ kg/m}^3}{8000 \text{ kg/m}^3}}{1 - \frac{1.2 \text{ kg/m}^3}{r_{AI}}} \quad \text{Equation 18}$$

The conventional mass M of a weight is not the mass of the weight itself, but rather is equal to the mass of the reference weight (standard mass) which under certain defined conditions¹ is in equilibrium with the weight being measured.

The air density ρ_a can be calculated from the conventional mass values given in the weight set for the determination of air density for the weights (designated the characteristic values of the weights), the material densities of the weights and the current weight values.

A number of Sartorius weighing instruments have the formulas for calculating the air density, including the values $\rho_{ST} = 8.000 \text{ g/cm}^3$ and $\rho_{AI} = 2.700 \text{ g/cm}^3$, integrated in their software. The current air density value can be saved and is then used to convert weight values to the actual masses of the samples weighed, using the formula derived at the beginning of this chapter:

$$m = W_v \cdot \frac{1 - \frac{r_a}{r_N}}{1 - \frac{r_a}{r_x}}$$

-
- ¹ Temperature $T = 20 \text{ }^\circ\text{C}$
 - Density of the standard mass at $20 \text{ }^\circ\text{C}$: $\rho_N = 8000 \text{ kg/m}^3$
 - Air density $\rho_a = 1.2 \text{ kg/m}^3$

Density Determination using Pycnometers

A pycnometer is a glass or metal container with a precisely determined volume, used for determining both the density of liquids and dispersion by simply weighing the defined volume (see also next chapter), but especially for determination of the density of powders and granules. Pycnometers can also be used in determining the density of the solid phase in a porous solid, but the sample must first be crushed or ground to the point where all pores are opened.

The pycnometers used in different areas of application have different shapes and standards. During measurement, it is important to make sure that all weighing operations are performed at a constant temperature and that there is no air trapped either in the liquid or between the sample particles.

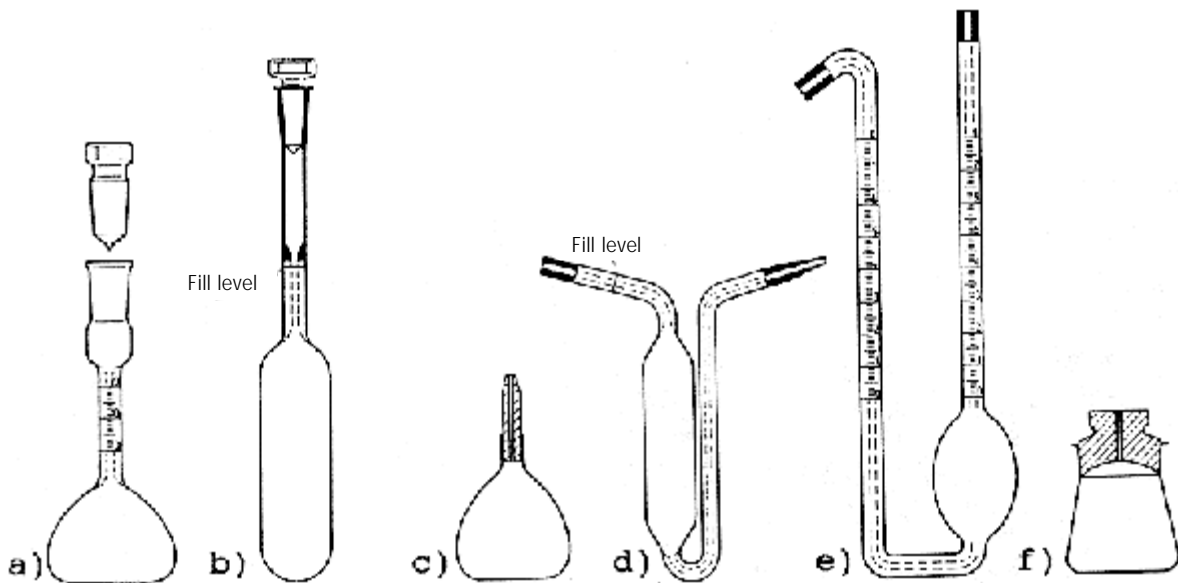


Figure 11: Different glass pycnometers for density determination in laboratories: The pycnometers made according to Gay-Lussac, DIN 12 797 (c) and to Hubbard, DIN 12 806 (f) are used for determining the density of solids; the volume indicated applies to complete filling after the stopper is inserted. The pycnometers made according to Bingham, DIN 12 807 (b) and to Sprengel, DIN 12 800 (d) have a line marking the fill level for the defined volume; the Reischauer, DIN 12 801 (a) and Lipkin, DIN 12 798 (e) pycnometers are marked with scales for checking the fill level.

Weighing a Defined Volume ("Weight per Liter ")

An especially simple gravimetric method for determining the density of flowing substances (liquids, powders, disperse systems) is to weigh a sample with a defined volume. In this case, the sample is placed in a container that has a defined volume, and the mass of the sample (after taring) is determined by weighing. The density can easily be determined according to $\rho = m/V$.

Different standardized containers are available for this purpose in different branches of industry; for example, a spherical 1 l-container for determining the density of cast material (slips) in the ceramic industry. In the lime industry, the tap density of unhydrated lime granules is determined using a standardized procedure, in which both the container for the sample and the procedure for filling the container are precisely defined. US and British standards call for the use of cylindrical stainless steel containers, called specific gravity cups, with various volumes and error margins.

Pycnometer Method

The pycnometer method is a very precise procedure for determining the density of powders, granules and dispersions that have poor flowability characteristics. The pycnometer method is more labor-intensive and far more time-consuming than the buoyancy and displacement methods.

This method also entails the difficulty of precise volume determination of a powder sample V_s for density determination of the solid ρ_s . The need for explicit determination of the volume of powders or granules can generally be avoided by performing 3 weighing operations and using an „auxiliary“ liquid with a known density.

$$r_s = \frac{m_s}{V_s} \quad \text{Equation 19}$$

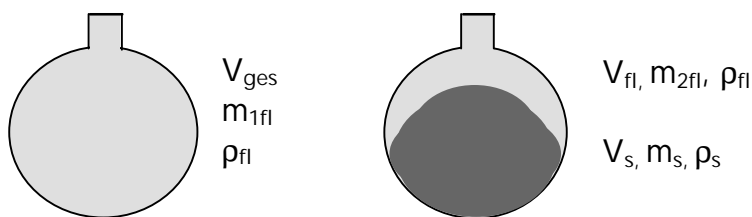


Figure 12: Pycnometer with contents

The volume of the solid V_s can only be determined indirectly:

$$V_s = V_{ges} - V_{fl} \quad \text{Equation 20}$$

The procedure is as follows:

- First the pycnometer is completely filled with liquid, and the mass of the liquid in the pycnometer determined. Once this value has been determined, the volume of the pycnometer V_{ges} is known.

$$V_{ges} = \frac{m_{1fl}}{r_{fl}} \quad \text{Equation 21}$$

- Next (after the pycnometer is emptied, cleaned, dried and brought to the required temperature) the pycnometer is filled to about 2/3 with sample material; this yields the mass of the powder m_s .
- The next step is to fill the pycnometer the rest of the way with liquid and weigh it again, which gives the combined mass of the sample with the liquid $m_{(fl+s)}$.

The mass of the liquid m_{2fl} can be calculated from this data

$$m_{2fl} = m_{(fl+s)} - m_s \quad \text{Equation 22}$$

which also yields the volume V_{fl} of the liquid in the pycnometer filled with water and liquid

$$V_{fl} = \frac{m_{2fl}}{r_{fl}} = \frac{m_{(fl+s)} - m_s}{r_{fl}} \quad \text{Equation 23}$$

The volume of the powdered sample V_s , the value actually sought, is yielded by the difference between the total volume V_{ges} and the volume of the liquid V_{fl} .

$$V_s = V_{ges} - V_{fl}$$

$$V_s = \frac{m_{1fl}}{r_{fl}} - \frac{m_{(fl+s)} - m_s}{r_{fl}}$$

Equation 24

Using the volume V_s in the original equation $\rho_s = m_s/V_s$ results in the following conversion²

$$\rho_s = r_{fl} \cdot \frac{m_s}{m_{1fl} - m_{(fl+s)} + m_s} \quad \text{or} \quad \rho_s = r_{fl} \frac{m_2}{m_1 + m_2 - m_3}$$

Equation 25

with the masses m_1 , m_2 and m_3 in the order of the procedural steps:

m_1 mass of the liquid in the pycnometer filled completely with liquid

m_2 mass of the sample material

m_3 mass of the sample and liquid together in the pycnometer

Thus this procedure represents another method for determining density "via detours," i.e. using a series of mass determination measurements.

$$^2 \rho_s = \frac{m_s}{V_s} = \frac{m_s}{\frac{m_{1fl}}{r_{fl}} - \frac{m_{(fl+s)} - m_s}{r_{fl}}} = \frac{m_s}{\frac{m_{1fl} - (m_{(fl+s)} - m_s)}{r_{fl}}} = \frac{r_{fl} \cdot m_s}{m_{1fl} - (m_{(fl+s)} - m_s)}$$

Other Methods of Density Determination

There are also other methods of density determination that are based on the Archimedean Principle. The density of air can be determined using two solid bodies of different densities (e.g., 2 weights made of different metals).

Density can also be determined by radioactive absorption by the material being tested. The absorption of the radiation will depend on the mass absorption coefficient, thickness of layers and density of the material. Once the mass absorption coefficient and thickness as well as the physical interrelationships are known, the density of the substance can be calculated.

On magnetic samples, the magnetic forces can also be utilized in density determination of solids or liquids.

Oscillation Method

The oscillation method is widely used to determine the density of homogenous liquids. This procedure is not suitable for use with suspensions or emulsions which, because they are made up of different phases, could separate.

The sample being tested is placed in a measuring chamber (usually a U-shaped glass tube) and mechanically vibrated. Calculation of the density uses the physical interrelationship between the frequency of the oscillation and the mass of the oscillation channel (the U-shaped tube with the sample). The equipment must be calibrated using liquids that have a known density and a viscosity similar to that of the sample material.

Suspension Method

The suspension method makes use of the Archimedean Principle in the special case of suspension in which the densities of the liquid and of the suspended solid are equal.

The density of the solid body can be determined by setting the density of the test liquid so that the sample body reaches a state of suspension. The density setting of the test liquid can also be achieved by mixing two liquids of different densities; the density of the sample is then determined from the proportions of the liquids mixed, or with the oscillation method (see page 18) or the displacement method.

Density Gradient Column

With a density gradient column, two liquids of different densities are layered in a glass tube so that over time, diffusion results in a vertical density gradient (a continuous change of the density throughout the height of the column). Small solids of various densities are then suspended at various heights, with each height indicating a particular density. Colored glass beads of known densities are available for calibration.

In addition to the density of small bodies (such as fibers, powder particles, small pieces of metal or plastic foil) this method can also be used to determine the density of drops of liquid—of course the liquid tested should be insoluble in the test liquid.

Schlieren Method

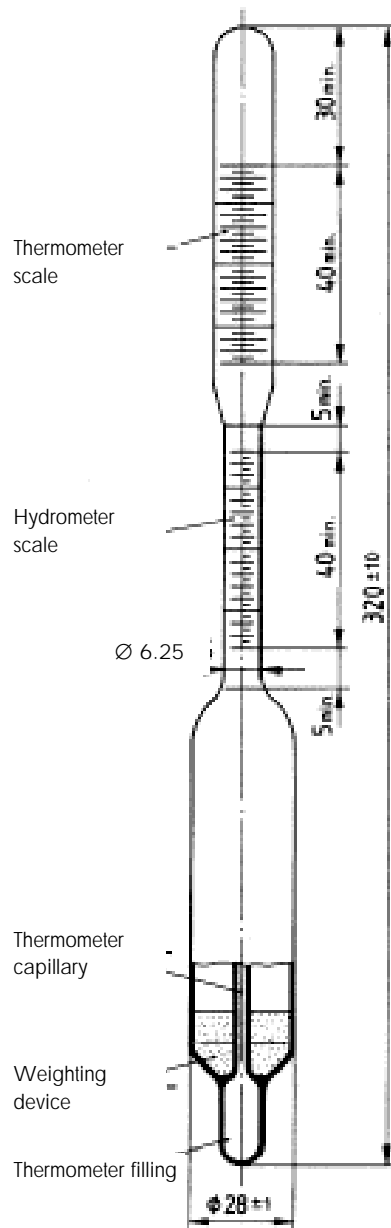
If you fill a capillary tube with liquid and hold it horizontally immersed in another liquid, the liquid will only flow horizontally from the tube if the densities of the two liquids are equal. If the density of the liquid flowing from the tube is lower or higher than that of the liquid in which the tube is immersed, schlieren (streaks) will form, flowing upward in the former case and downward in the latter.

Hydrometers

Hydrometers, also known as spindles, are simple measuring instruments for determining the density of liquids or dispersions. They are forms of **plummet** that float on the surface and then sink to a certain level, depending on the density of the liquid. The density of the liquid can be determined from the depth the plummet sinks (from the volume of the displaced liquid) by comparing the height of the liquid in the container to the scale marked on the hydrometer. For certain applications, there are also hydrometers that show the concentration of a given substance in an aqueous solution; for instance; sugar (in a saccharimeter), alcohol (in an alcoholometer), battery acid or anti-freeze.

Thermohydrometer for use with milk and skimmed milk

Thermometer scale



Hydrometer scale

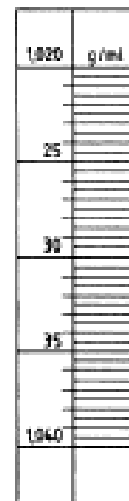


Figure 13: Special hydrometer with integrated thermometer in accordance with DIN 10 290 for determining the density of milk and skimmed milk – the density is dependent on the fat content of the milk.

Practical Applications

Determining the Density of Solids

Characteristic Features of Sample Material

Solid bodies retain their volume and shape under atmospheric pressure. Examples of solids for which it can be useful to know the density include metals, glass and plastics. These solids may be made up of only one or many phases; one phase may also be embedded in another (for instance, in fiberglass-reinforced plastic) or the different phases may be interlocking, such as the many small crystals in a homogenous metallic material.

An important factor in choosing a suitable sample for density determination is the question of whether the density is required as a characteristic of a material or whether density determination is performed to check for defects in a material. The choice of procedure for density determination will depend on this factor as well.

Choosing a Density Determination Method

The best procedures for density determination on solids are the buoyancy and displacement methods, both of which are based on the Archimedean Principle. Prerequisite for these methods is the use of a liquid for buoyancy that does not react with the sample material, but wets it thoroughly.

The suspension method, for example, is widely used in the glass processing industry. Glass samples are placed in an organic liquid in which they float at room temperature. Because the density of the liquid is 100 times more temperature-dependent than that of glass, the glass can be brought to the point where it is suspended within the liquid by slowly increasing the temperature in the test system; in this way, the density of the glass can be determined.

Performing Density Determination using the Displacement Method

Equipment Required

- Weighing instrument
- Thermometer
- Stand with holding device for samples
- Beaker with liquid for buoyancy that has a known density – distilled water for all materials that do not react with water

Preparation of the Sample, Testing Procedure and Evaluation

The beaker is placed on the pan of the balance and the sample-holding device is immersed in the liquid, to the same depth that it will later be immersed with the sample on it. The weighing instrument is tared.

The sample is placed next to the beaker on the weighing pan. The mass of the sample in air $m(a)$ is determined.

The sample is placed in the holding device on the stand and immersed in the liquid. The weight readout shows the mass of the displaced liquid m_{fl} .

The density of the sample ρ is calculated according to $\rho = \rho_{fl} \cdot \frac{m_{(a)}}{m_{fl}}$.

Determining the Density of Porous Solids

Characteristic Features of Sample Material

There are a number of terms used in connection with the density of porous materials, such as solid density, true density, bulk density, apparent porosity, open porosity and closed porosity.

Porous solids consist of one or more solid phases and pores. Pores are cavities filled with air (or other gas). These openings are found either between individual crystals in the solid material, or as gas bubbles in glass phases; i.e., solidified in a non-crystalline form. Thus there are basically two forms of pores: open and closed. Among open pores, in turn, there are also different types; for instance, there are pores through which liquid may flow, and saturable pores. With these designations, the type of soaking medium and other conditions must be given (e.g., water at a temperature of 22°C and pressure of 2500 Pa).

The term "pore" is used for openings or gaps from 1 nm to 1 mm. Openings larger than 1 mm are referred to as cracks or voids; those under 1 nm are defects in the crystal lattice. Pores are an important element in the micro structure of many materials; the quantity, type, shape, orientation, size and size distribution of pores significantly affect many important characteristics of a material; for example, the frost-resistance of roof tiles, or insulating properties of lime malm bricks or porous concrete. Such characteristics as mechanical solidity and corrosion resistance are also affected by pores in a material.

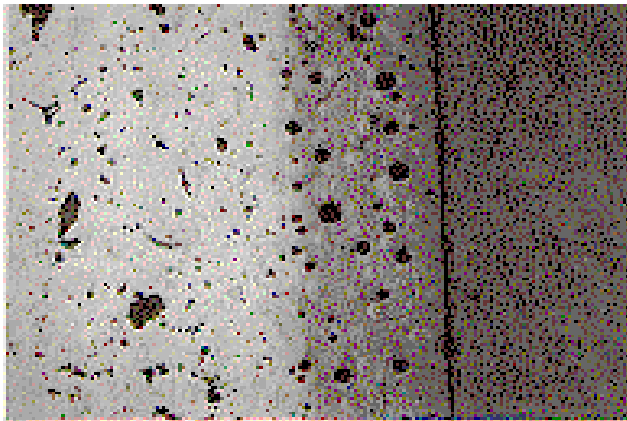


Figure 14: Microstructure of a porcelain plate. Magnification: approximately 80x. Left: porcelain with irregular pores between phases; right: glaze layer melted during firing, with closed spherical pores (bubbles); extreme right: synthetic resin as embedding medium for the polishing preparation

The density determination procedure will depend on whether the value sought is the density of the solid matter only or the density of the material including pores; after all, it may be important to know the porosity of the material.

The density of the solid material (not the solid body), which used to be termed the "true density" is now simply referred to as "density": $\rho_t = m / V_{solid}$. The pores are not included in this measurement.

"Bulk density" is the quotient of mass and total volume of a sample: $\rho_b = m / V_b$. The bulk density is an average of the density of the solid and the gas found in the pores.

"Open porosity" is the ratio of the volume of open pores to the total volume of the porous body, in percent: $\pi_a = V_a / V_b$.

"Closed porosity" is the ratio of the volume of closed pores to the total volume of the porous body, in percent: $\pi_f = V_f / V_b$.

The apparent porosity is the ratio of the volume of all pores to the total volume of the material, in percent: $\pi_t = V_t / V_b$. The apparent porosity is the sum of open and closed porosity: $\pi_t = \pi_a + \pi_f$.

Choosing a Density Determination Method

Both the buoyancy and the displacement methods are suitable for density determination on porous solids. The solid density can also be determined using the pycnometer method, by first grinding the sample until the grain size is roughly equal to the pore size.

To determine the bulk density of porous material, the sample can also be covered in a wax or latex coating or layer to prevent liquid from entering open pores (see for example the German Industry Standard (DIN) 2738). Density determination can then be performed using the buoyancy method.

Performing Density Determination using the Buoyancy Method (in Accordance with European Standard EN 993-1)

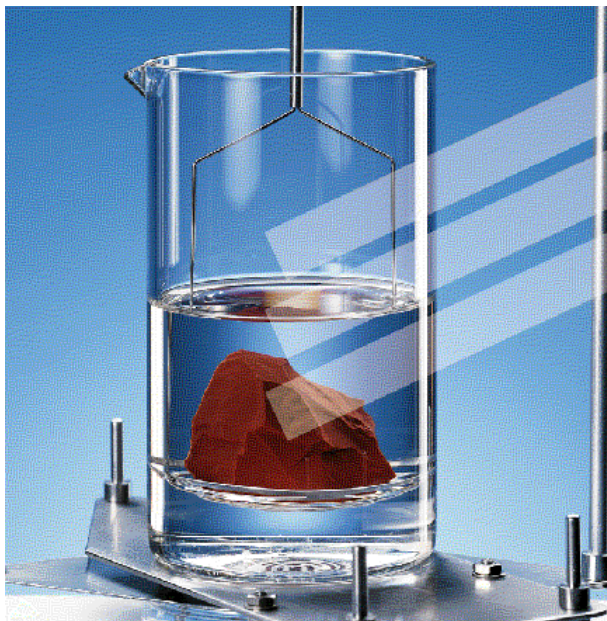


Figure 15: Mode of procedure for density determination using the buoyancy method and the Density Determination Set from sartorius

Equipment Required

- Drying oven; temperature: 110 ± 5 °C
- Weighing instrument: margin of error: 0.01 g
- Frame to be placed over the weighing pan: included with the Density Determination Set
- Vacuum generator with adjustable pressure and pressure gauge
- Thermometer with an error margin of 1 °C
- Liquid for saturation – distilled water for all materials that do not react with water

- Dessicator.

Preparation of the Sample

Shape and size (total volume between 50 cm³ and 200 cm³) of the sample, as well as the number of samples to be tested, are defined in the Standard.

Testing Procedure and Evaluation

The sample is first dried in the drying oven until it reaches a constant mass and then cooled to room temperature in the dessicator. The mass of the sample is then determined in air using the weighing instrument. → m₁

The sample is then evacuated under precisely defined conditions and saturated (in the vacuum) until the open pores – as stated in the test specifications – are filled with the saturation liquid. The apparent mass of the saturated sample is then determined using a hydrostatic balance (or using the Density Determination Set). The sample must be completely immersed in a beaker filled with the saturation fluid for buoyancy. → m₂

The temperature of the saturation liquid must be determined.

Then the mass of the saturated sample is determined by weighing in air. Liquid that remains on the surface of the sample must be removed with a damp sponge before weighing. The weighing operation must be performed quickly, to avoid loss of mass due to evaporation. → m₃

The density of the saturation liquid ρ_{fl} must be measured or taken from a table of density values at defined temperatures.

The bulk density ρ_b in g/cm³ is calculated as follows:

$$r_b = \frac{m_1}{m_3 - m_2} \cdot r_{fl} \quad \text{Equation 26}$$

The open porosity π_a in volume percent is calculated as follows:

$$p_a = \frac{m_3 - m_1}{m_3 - m_2} \cdot 100 \quad \text{Equation 27}$$

The apparent porosity π_t is calculated as follows:

$$p_t = \frac{r_t - r_b}{r_t} \cdot 100 \quad \text{Equation 28}$$

The apparent porosity is the sum of open and closed porosity (π_t = π_a + π_f); thus it follows that for the closed porosity π_f:

$$p_f = p_t - p_a \quad \text{Equation 29}$$

- m₁ Mass of the dried sample
- m₂ Apparent mass of the saturated sample weighed in liquid
- m₃ Mass of the saturated sample weighed in air

r_t	Density of the solid, determined according to EN 993-2 (or calculated from the composition)
r_{fl}	Density of the fluid for buoyancy
r_b	Bulk density of the sample

One of the numeric values often given for ceramics for the open porosity, in addition to the values listed above, is the water absorption. The water absorption in percent is yielded by the difference in mass between the saturated sample and the dried sample, relative to the mass of the dried sample. The resulting figure can be used in dividing ceramics into "dense" and "porous" grades.

Additional information about the type and size distribution of the pores can be gained using a mercury porosimeter: The porous samples are put under pressure with mercury, whereby the pressure is increased at certain stages so that, depending on the pore diameters, a certain number of the pores are filled with mercury. This can yield information on the proportion and diameter of open pores that are accessible from the outside.

Another method for determining number, shape and size of pores is image analysis, the quantitative statistical evaluation of polished sections under a microscope, similar to the image shown in Figure 14.

Determining the Density of Powders and Granules

Characteristic Features of Sample Material

The term **powder** refers to "a heap of particles, usually with dimensions smaller than 1 mm."

Granules are larger particles than those that make up a powder. The term "granules" has different meanings in different areas of application:

- Material made up of "secondary" particles, which in turn are made up of agglomerated particles of a fine powder; or
- Material that was heated to the melting point and then cooled very quickly, causing it to take on a teardrop shape; for example, intermediate products in the plastics or porcelain enamel industries.

Choosing a Density Determination Method

The pycnometer method is the only method that can be used with powders or granules.

Performing Density Determination using the Pycnometer Method (in Accordance with German and European Standard DIN EN 725-7)



Figure 16: Pycnometer with integrated thermometer

Equipment Required

- Distilled water and another liquid, such as ethanol
- Pycnometer with thermometer and sidearm with polished glass stoppers
- Water bath
- Vacuum pump
- Weighing instrument; error margin: 0.0001 g

Testing Procedure and Evaluation

The pycnometer must be carefully cleaned and dried; it is then filled with distilled water, evacuated under precisely defined conditions, and brought to within ± 0.1 K of the required temperature in a water bath. Then the pycnometer is filled. The volume of the pycnometer is calculated from the mass of the water at the test temperature: $V_{\text{pycnometer}} = m_{\text{water}} / \rho_{\text{water}}$.

The pycnometer is then dried, filled with ethanol and weighed, following the same procedure as that described above. The density of the ethanol at the test temperature can be calculated from the mass of the ethanol and the volume of the pycnometer:

$$\rho_{\text{ethanol}} = m_{\text{ethanol}} / V_{\text{pycnometer}}$$

$$m_{\text{ethanol}} = m_1.$$

Once the pycnometer has been cleaned and dried again³, it is loaded with about 10 g (at a density between 2.5 and 4 g/cm³) of the powder, which has been dried at a temperature 10 K below the decomposition point of the powder. $\rightarrow m_2$

Enough ethanol is now added to the pycnometer to wet the powder; the pycnometer is then evacuated and shook to release as many air bubbles as possible. More ethanol is added to the pycnometer; after this has been heated to the test temperature, the pycnometer is completely filled. The total mass of powder and ethanol is determined. $\rightarrow m_3$

The data is evaluated using the formula $\rho = \rho_{\text{ethanol}} \cdot \frac{m_2}{m_1 + m_2 - m_3}$, the density of the sample material is given with a precision of 0.001 g/cm³.

Determining the Density of Homogenous Liquids

Characteristics of Sample Material

Homogenous liquids are relatively simple systems; unlike dispersions, they are always single-phase systems. When substances are mixed, such as water with alcohol or sugar with water, this is referred to as the solution of one substances in the other. A genuine solution is clear, the particles of the dissolved substance are present as molecules or ions in the solution.

³ The latest weighing instruments from sartorius come equipped with density determination software that eliminates with time-consuming steps for drying in the drying oven and cooling in the dessicator; the program has a second tare memory which can be used to tare the weight of water remaining in the pycnometer. This considerably simplifies work in the laboratory.

The density of a solution depends on the concentration of the dissolved substance; in other words, when the interrelationships are known, the density value can be used to derive the concentration of the solution.

At 20°C most fluids have a density between 600 kg/m³ and 2000 kg/m³ or 0.6 g/cm³ to 2.0 g/cm³. The density of fluids is far more temperature-dependent than that of solids. This means that the temperature must **always** be monitored carefully and, if necessary, the sample heated or cooled to the required temperature.

Choosing a Density Determination Method

There are several methods that can be used to determine the density of liquids, including the hydrometer, pycnometer, oscillator, buoyancy and displacement methods. The choice of method depends, among other things, on the degree of precision required and the amount of sample material available.

Performing Density Determination using the Buoyancy Method

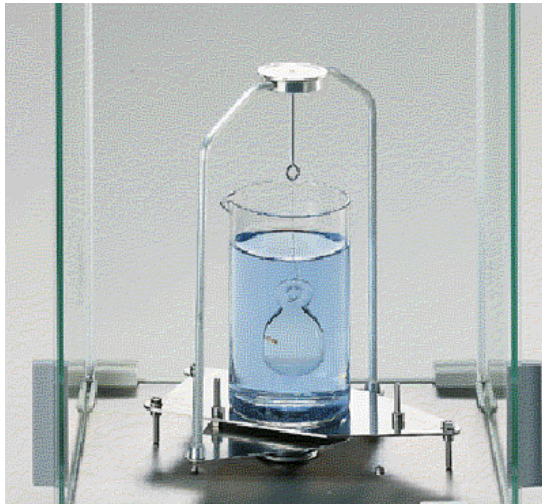


Figure 17: Determining the density of a liquid using the buoyancy method

Equipment Required

- Weighing instrument
- Density Determination Set
- Plummet with known volume (10 cm³ in the sartorius Density Determination Set; see Figure 17)
- Thermometer
- In some cases: water bath for adjusting the temperature of the sample

Preparation of the Sample, Test Procedure and Evaluation

Position an empty beaker on the bridge and hang the plummet from the frame provided in the Density Determination Set.

Tare the weighing instrument with the plummet.

Fill the beaker with the liquid to be tested up to a level 10 mm higher than the plummet.

The negative value shown on the weight readout corresponds to the buoyancy of the plummet in the liquid.

The density of the liquid is calculated by dividing the measured value by the volume of the plummet

$$r = \frac{m_{fl}}{V_{TK}}$$

Determining the Density of Dispersions

Characteristics of Sample Material

Disperse systems or **dispersions** are combinations of two or more phases, each of which is insoluble in the other(s). One phase, called the dispersion medium, is always contiguous, while the other phase or phases are present in the medium in the form of finely distributed isolated particles.

In a colloid dispersion, the particles are generally between 1 μm and 1 nm in size. If the particles are larger than $> 1 \mu\text{m}$, this is referred to as a coarse dispersion; if particles are smaller than $< 1 \text{ nm}$, it is a molecular dispersion.

There are many examples of dispersions, because "**dispersion**" is the generic term for all systems, independent of the state of the phases. Different types of dispersions include:

- Suspensions – Mixtures of solid particles in a liquid
Examples: "Dispersion" paint, ceramic slips, abrasive liquid cleanser, toothpaste, ink, etc.
- Emulsions – Mixtures of two liquids that are mutually insoluble, where one is present in the form of finely distributed minuscule drops in the other
Examples: Cremes, lotions, mayonnaise, milk, the classic oil-and-vinegar salad dressing, etc.
- Foams – Mixtures of gas bubbles in a liquid (or a solid)
- Mist – Mixtures of small drops of liquid in a gas phase
- Smoke – Mixtures of solid particles in a gas phase

The term "stability" in reference to dispersion is somewhat problematic, because these are actually unstable systems. This can be seen in their tendency to separate. The terms "stable suspension" and "stable emulsion" are often used to refer to systems that remain constant over a certain period of time.

Choosing a Density Determination Method

Many of the same methods used on liquids or solids can also be used for determining the density of dispersions. The best choice for a given sample material will depend on the consistency of the sample.

The oscillation method is not well-suited for use here, for a number of reasons. The many phase boundaries are a disadvantage; the viscosity has an effect on the measurement, and the vibration during measurement can promote separation of the phases, which means the values obtained will not be representative of the overall average.

Hydrometers can be used, but it must be ensured that the suspension or emulsion does not show signs of separating.

The pycnometer can also be used on dispersions, just as with density determination on liquids or powders. Different containers are used in different branches of industry; the containers may be filled to the rim or up to a marking with the sample material and weighed. The potential separation of the sample phases during measurement is not a problem with this procedure.

The buoyancy or displacement method can be used for many suspensions and emulsions. In this case, too, care must be taken to avoid separation of the sample phases; the flow behavior of the suspension must also allow the plummet to sink quickly.

Performing Density Determination using the Displacement Method

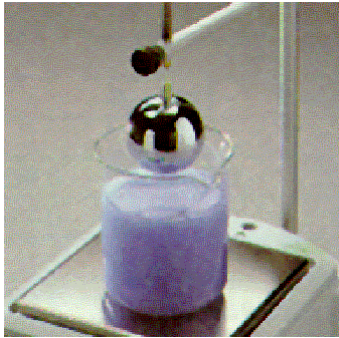


Figure 18: Density determination using the displacement method, with a gamma sphere as a plummet, affixed to a holder mounted next to the weighing instrument

Equipment Required

- Weighing Instrument
- Plummet with known volume
- Holder
- Thermometer
- In some cases: water bath for adjusting the temperature of the sample

Preparation of the Sample, Test Procedure and Evaluation

Bring the sample to the required temperature and place it in a beaker. Place the beaker on the weighing pan and tare the weighing instrument. Immerse the plummet in the test substance up to the marking.

The weight readout shows the mass of the displaced liquid directly (see Page 12). The density of the sample is determined by dividing the measured value by the volume of the plummet $\rho = \frac{m_{fl}}{V_{TK}}$.

Errors in and Precision of Density Determination

In the two previous chapters (Fehler! Verweisquelle konnte nicht gefunden werden. and Fehler! Verweisquelle konnte nicht gefunden werden.) the fundamentals of these two hydrostatic density determination methods were explained and the formulas for calculation of the density were derived.

If a high degree of precision is required, the existing conditions must be allowed for. Technically speaking, the weighing instrument does not show the mass of the samples—the value given in the equations—but rather the weight value for the samples in air. For more precise results, use **the weight values obtained after air buoyancy correction**.

When using the buoyancy method, the immersion level of the pan hanger assembly is affected when the sample is immersed, which produces **additional buoyancy**. This must also be considered in more precise calculations.

In general, density determination procedures require very careful work; it is especially important to make sure the temperature remains constant during testing.

Bubbles entering the liquid when the sample is immersed can also affect results; bubbles adhering to the test piece will distort the measurement results.

Air Buoyancy Correction

For high-precision density determination, it is important to note that the weighing instrument does not directly determine the mass of the sample, but its **weight value**. This value is dependent on the **air density**, which in turn depends on the pressure and temperature and must be corrected by the value for air buoyancy.

Between the mass of a solid body m and its weight value in air W ; that is, under consideration of the air buoyancy on the sample, the following relationship is generally valid (ρ_G = density of the standard):

$$m = W_v \cdot \frac{1 - \frac{\rho_a}{\rho_G}}{1 - \frac{\rho_a}{\rho}} \quad \text{Equation 30}$$

Displacement Method

If you include this equation in the calculation of the density when the displacement method is used, the equation for determining the density of the solid body follows with **allowance for the air buoyancy** (see Appendix, page 58, for derivation).

$$\rho_s = \rho_{fl} \cdot \frac{m_s}{m_{fl}} = (\rho_{fl} - \rho_a) \cdot \frac{W_s}{W_{fl}} + \rho_a \quad \text{Equation 31}$$

This is the manner in which density is calculated by the software that comes with sartorius weighing instruments.

Buoyancy Method

If you include the equation $m = W_v \cdot \frac{1 - \frac{r_a}{r_G}}{1 - \frac{r_a}{r}}$ for $m_{(a)}$ and $m_{(fl)}$ in the equation for density

determination using the buoyancy method $r_s = r_{fl} \cdot \frac{m_{(a)}}{m_{(a)} - m_{(fl)}}$, mathematical conversion results in

$$r_s = (r_{fl} - r_a) \cdot \frac{W_{(a)}}{W_{(a)} - W_{(fl)}} + r_a \quad \text{Equation 32}$$

the formula for calculating the density of solid bodies with allowance for air buoyancy.

This is the manner in which the air buoyancy is accounted for in the software that comes with sartorius weighing instruments (for details, see the operating instructions for the instrument in question). Moreover, another correction factor is included in the calculation which takes into account the additional buoyancy caused by the immersion of the wires on the pan hanger assembly (see below).

Pycnometer Method

The formula for calculating density with the air buoyancy correction using the pycnometer method is:

$$r_s = (r_{fl} - r_a) \cdot \frac{W_2}{W_1 + W_2 - W_3} + r_a \quad \text{Equation 33}$$

This is the formula used by the software that comes with sartorius weighing instruments.

Air Buoyancy Correction for the Pan Hanger Assembly

Buoyancy Method

For high-precision measurement, another consideration besides the air buoyancy is the **additional buoyancy of the pan hanger wires**, caused by the fact that the **height of the liquid is increased when the sample is immersed** which means the wires are deeper under the surface than without the sample. (The buoyancy of the pan hanger assembly is not included in the density calculation if the weighing instrument is tared with the pan hanger assembly.)

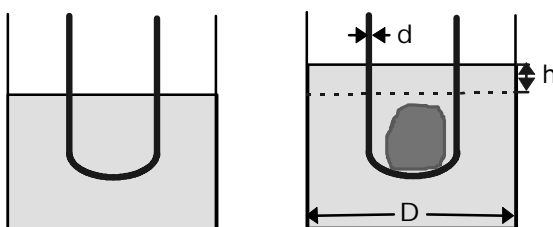


Figure 19: Diagram illustrating the calculation of the buoyancy caused by increased height of liquid when sample is immersed

With the procedure for using the buoyancy method with the Density Determination Set, the wires of the pan hanger assembly are deeper under the surface of the water when the sample is immersed

because the volume of the sample causes more liquid to be displaced. Because more of the wire is immersed, it causes more buoyancy; the additional buoyancy can be calculated and corrected for in the results.

The amount of volume by which the liquid increases in a container with diameter D corresponds to the volume V_{fl} in Figure 1

$$V_{fl} = \frac{\rho \cdot D^2}{4} \cdot h. \quad \text{Equation 34}$$

The volume V_s of the sample is

$$V_s = \frac{m_{(a)} - m_{(fl)}}{\rho_{fl}} \quad \text{Equation 35}$$

These two volumes are identical, $V_{fl} = V_s$. Using the above equations and solving for h, the increase in height of the liquid, yields

$$h = \frac{(m_{(a)} - m_{(fl)}) \cdot 4}{\rho_{fl} \cdot \rho \cdot D^2} \quad \text{Equation 36}$$

The buoyancy force F_{BD} exerted on two wires of diameter d at the fluid level h is

$$F_{BD} = \rho_{fl} \cdot V_D \cdot g = \rho_{fl} \cdot (2 \cdot \frac{\rho \cdot d^2}{4} \cdot h) \cdot g \quad \text{Equation 37}$$

and with the value for h included yields

$$\begin{aligned} F_{BD} &= \rho_{fl} \cdot 2 \cdot \frac{\rho \cdot d^2}{4} \cdot \frac{(m_{(a)} - m_{(fl)}) \cdot 4}{\rho_{fl} \cdot \rho \cdot D^2} \cdot g \\ F_{BD} &= \frac{\rho_{fl} \cdot 2 \cdot \rho \cdot d^2 \cdot (m_{(a)} - m_{(fl)}) \cdot 4 \cdot g}{4 \cdot \rho_{fl} \cdot \rho \cdot D^2} \\ &= 2 \cdot \frac{d^2}{D^2} \cdot (m_{(a)} - m_{(fl)}) \cdot g \end{aligned} \quad \text{Equation 38}$$

This means that the buoyancy caused by the wires is proportional to the relation between the diameters of wires and beaker.

In addition to the buoyancy of the sample – which is to be determined – the measured value also includes part of the "wire buoyancy;" thus the wire buoyancy must be subtracted from the measured value to yield the buoyancy of the sample alone $F_{BS(corr)}$:

$$\begin{aligned} F_{BS(corr)} &= \left[(m_{(a)} - m_{(fl)}) - \left(2 \frac{d^2}{D^2} \cdot m_{(a)} - m_{(fl)} \right) \right] \cdot g \\ F_{BS(corr)} &= \underbrace{\left(1 - 2 \frac{d^2}{D^2} \right)}_{\text{correction factor}} \cdot (m_{(a)} - m_{(fl)}) \cdot g \end{aligned} \quad \text{Equation 39}$$

When the diameters of the wires and the beaker are known, the factor by which the value measured for buoyancy must be multiplied can be calculated. In the Density Determination Set

from sartorius, the wire diameter is $d = 0.7$ mm, the beaker diameter $D = 76$ mm, and the pan hanger assembly has two wires. Thus the correction factor is:

$$\text{Corr} = 1 - 2 \frac{d^2}{D^2} = 1 - 2 \frac{0.7^2}{76^2} = 0.99983.$$

The smaller the diameter d of the wires, the larger the diameter D of the beaker and the fewer wires on the pan hanger assembly, the nearer the correction factor will be to 1; i.e., the correction is negligible. These conditions are easy to create when performing density determination using the below-scale or below-balance weighing method.

Returning to the formula for density determination using the buoyancy method: For the buoyancy correction for the wires, the equation $r_s = r_{fl} \cdot \frac{m_{(a)}}{m_{(a)} - m_{(fl)}}$ yields

$$r_s = r_{fl} \cdot \frac{m_{(a)}}{[m_{(a)} - m_{(fl)}] \cdot \text{Corr}}$$

or, when the air buoyancy is also accounted for,

$$r_s = (r_{fl} - r_a) \cdot \frac{W_{(a)}}{(W_{(a)} - W_{(fl)}) \cdot \text{Corr}} + r_a \quad \text{Equation 40}$$

This is the formula used for density determination in the software that comes with sartorius weighing instruments. The weighing instruments can work with the preset values for a standard air density of $\rho_a = 0.0012$ g/cm³ and a correction factor of 0.99983 for the sartorius Density Determination Set. Alternatively, user-defined correction factors can be entered.

Displacement Method

Errors caused by the additional buoyancy when the pan hanger assembly for the sample or lines or wires of the pan hanger can be eliminated at the outset by immersing the pan hanger assembly just as deep in the liquid when it is weighed empty or tared as it will be later with the sample on it.

In addition, the test conditions can be arranged to make the correction factor ≈ 1 ; i.e., by using large container diameter, small pan hanger assembly diameter, only one pan assembly if possible. The equation for calculation of the solid body density using the displacement method – with allowance for air buoyancy and the correction factor for the sample hanger – is:

$$r_s = (r_{fl} - r_a) \cdot \frac{W_{(a)}}{W_{(fl)} \cdot \text{Corr}} + r_a \quad \text{Equation 41}$$

This is the formula used by the software that comes with sartorius weighing instruments. The default settings in this software include a numerical value of 1.0 for the correction factor and $r_a = 0.0012$ g/cm³ for the standard air density. User-defined values can also be entered.

When the displacement method for density determination on liquids is used, including on paints and varnishes, the plummet is usually made of metal (gamma sphere) and is tapered in one section (see Figure 20); the nominal volume of the plummet is calculated to the middle of the tapered portion. There are different sizes of plungers available for use with different degrees of surface

tension in the liquid being tested. The influence of the bulge of liquid around the stem of the plummet is also accounted for.

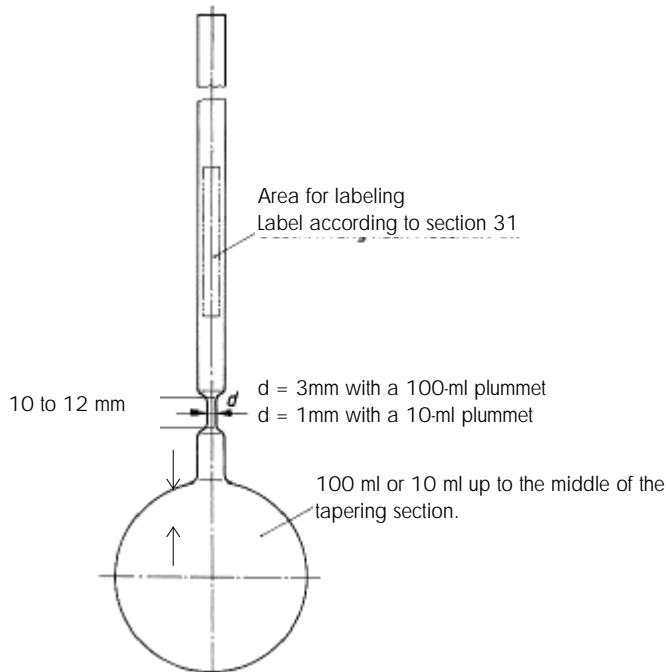


Figure 20: Plummet (in accordance with DIN 53 217 Part 3) for determining the density of paints, varnishes, and similar coating materials using the displacement method.

Prevention of Systematic Errors

Hydrostatic Method

To limit the errors in density determination with different hydrostatic procedures, the following should be observed:

- The temperature must be kept constant throughout the entire experimental procedure. With water as buoyancy medium, for example, a temperature variation of 0.1°C changes the density by 0.00002 to 0.00003 g/cm^3 ; with alcohol, by $\approx 0.0001\text{ g/cm}^3$.
- The measuring instrument should be loaded exactly in the center, to maximally limit off-center loading errors. When weighing in air using the Sartorius Density Determination Set with the sample on top of the frame, eccentric positioning of the sample due to the shape of the frame causes force to be conducted to two external points on the base of the frame resulting in a greater torque than with the use of "normal" weighing pans with the same deviation from the center.
- After submersion in the liquid there must be no air bubbles on the sample or on the pan hanger assembly. These cause an additional buoyancy and falsify the measured weight. To prevent this, one can wet the sample in a separate beaker or in an ultrasound bath.
- Errors due to adhesion of liquid to the wire of the pan hanger assembly can be prevented by taring the weighing instrument with submersed pan hanger assembly before measuring.

- **Air buoyancy** causes an error of density of $\approx 0.0012 \text{ g/cm}^3$ (corresponding with air density under normal conditions); therefore in calculating the density, the equation should take air buoyancy into account (see p. 31, "Air Buoyancy Correction").
- After submersing the sample in the container, the level of the liquid rises so that the **wires of the pan hanger assembly cause additional buoyancy**. Depending on the diameter of the beaker used and the number of wires of the assembly, this additional buoyancy can be corrected (see p. 32 Air Buoyancy Correction for the Pan Hanger Assembly.)

Pycnometer Method

To limit the errors in determining density with the pycnometer method, the following should be observed:

- The **temperature must be kept constant** throughout the entire experimental procedure; the temperature of the samples must be carefully controlled. With water as the liquid medium for example, a temperature change of 0.1°C changes the density by 0.00002 to 0.00003 g/cm^3 ; with alcohol, by $\approx 0.0001 \text{ g/cm}^3$.
- There should be no **air bubbles** in the liquid medium or on the sample.
- The **air buoyancy** causes an density error of $\approx 0.0012 \text{ g/cm}^3$ (corresponding to air density under normal conditions); therefore in calculating the density, the equation should take air buoyancy into account (see p. 33, "Air Buoyancy Correction").

When proper care is exercised using the pycnometer, this method can be used for highly accurate determination of the density of materials.

Error Calculation

With careful work and with the prevention of the above-mentioned systematic errors, the errors of density determination can be calculated according to the rules of error reproduction. The density error $\Delta\rho$ is based mainly on measuring results in mass determination.

Generally, for the determination of the total errors ΔF of a dimension that is calculated from several measuring values:

With the sum (and the difference) the **absolute single errors** add up quadratically:

$$\Delta F = \sqrt{\Delta F_1^2 + \Delta F_2^2 + \dots} \quad \text{Equation 42}$$

With products (and quotients) the **relative single errors** add up quadratically. (The relative error is the absolute error in relation to the measured value.):

$$\frac{\Delta F}{F} = \sqrt{\left[\frac{\Delta F_1}{F_1}\right]^2 + \left[\frac{\Delta F_2}{F_2}\right]^2 + \dots} \quad \text{Equation 43}$$

Buoyancy Method

For the density determination of solid bodies using the buoyancy method the following relationship

is applied
$$\mathbf{r}_s = \mathbf{r}_{fl} \cdot \frac{m_{(a)}}{m_{(a)} - m_{(fl)}} \quad \text{or} \quad \mathbf{r}_s = (\mathbf{r}_{fl} - \mathbf{r}_a) \cdot \frac{W_{(a)}}{(W_{(a)} - W_{(fl)}) \cdot \text{Korr}} + \mathbf{r}_a$$

Because the correction factor for the pan hanger assembly and the density of air have no noticeable effect on the error of density, they do not need to be regarded in error calculation.

If one uses the basic rules of error calculation, the absolute error of the denominator $\Delta[m_{(a)}-m_{(fl)}]$ is calculated next:

$$\Delta[m_{(a)} - m_{(fl)}] = \sqrt{\Delta m_{(a)}^2 + \Delta m_{(fl)}^2} \quad \text{Equation 44}$$

followed by the total relative error of density $\Delta\rho/\rho$

$$\frac{\Delta r}{r} = \sqrt{\left[\frac{\Delta r_{fl}}{r_{fl}}\right]^2 + \left[\frac{\Delta m_{(a)}}{m_{(a)}}\right]^2 + \left[\frac{\Delta(m_{(a)} - m_{(fl)})}{m_{(fl)}}\right]^2} \quad \text{Equation 45}$$

The total error of **weighing in air** ($\Delta m_{(a)}$) is the sum of **reproducibility** and a **linearity** error of one digit – regardless of the type of weighing instrument, because differential weighing is performed.

The maximum error of **weighing in liquids** ($\Delta m_{(a)} - \Delta m_{(fl)}$) is on average assumed to be 10 times as great as with weighing in air – this assumption is based on many measurements in density determination using the buoyancy method.

For the error of liquid density determination, the value of 0.00003 g/cm³ for water and 0.00009 g/cm³ for ethanol are applied; that is the value of a thermometer reading error of $\pm 0.1^\circ\text{C}$ which corresponds to a temperature variation during measurement of $\pm 0.1^\circ\text{C}$.

The following figures (see Figure 21: Solid density determination using the buoyancy method – the relative error of density is dependent on the sample size and the sample density) show the relative error of solid density determination with the buoyancy method in water or ethanol, depending on the sample size and the density of the sample examples, using sartorius weighing instruments with different readabilities and weighing capacities.

It is clear that the error in density determination is dependent on the density of the sample to a significant degree: **the lower the density of the sample, the greater the error of the end result**⁴.

A further figure (see figure 27) shows the error in density determination of liquids using the buoyancy method for liquid densities between 0.5 and 2.2 g/cm³ with the use of the glass plummet included with the sartorius Density Determination Set. The plummet has a volume of (10 + 0.01) cm³ and a density of 2.48 g/cm³ with a tolerance of 0.5 mg in relation to the buoyancy of water. Here too the error of density is dependent on the density value of the investigated sample.

⁴ For the calculation of $\frac{\Delta r}{r}$ $m_{(fl)} = m_{(a)} \cdot \left[1 - \frac{r_{fl}}{r_s}\right]$ is plugged in for $m_{(fl)}$;

for ρ_{fl} the density of water 1.0 g/cm³ is used, for the density of ethanol 0.789 g/cm³

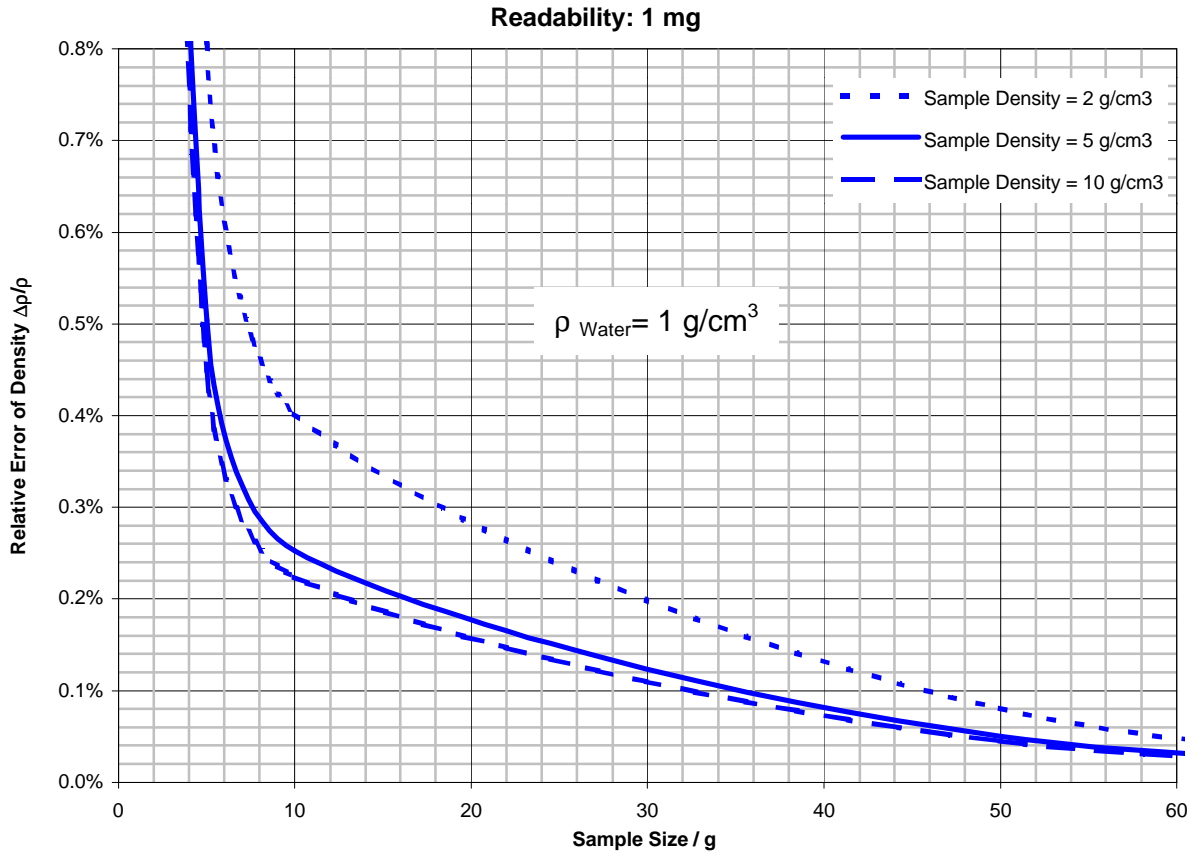


Figure 21: Solid density determination using the buoyancy method – the relative error of density is dependent on the sample size and the sample density

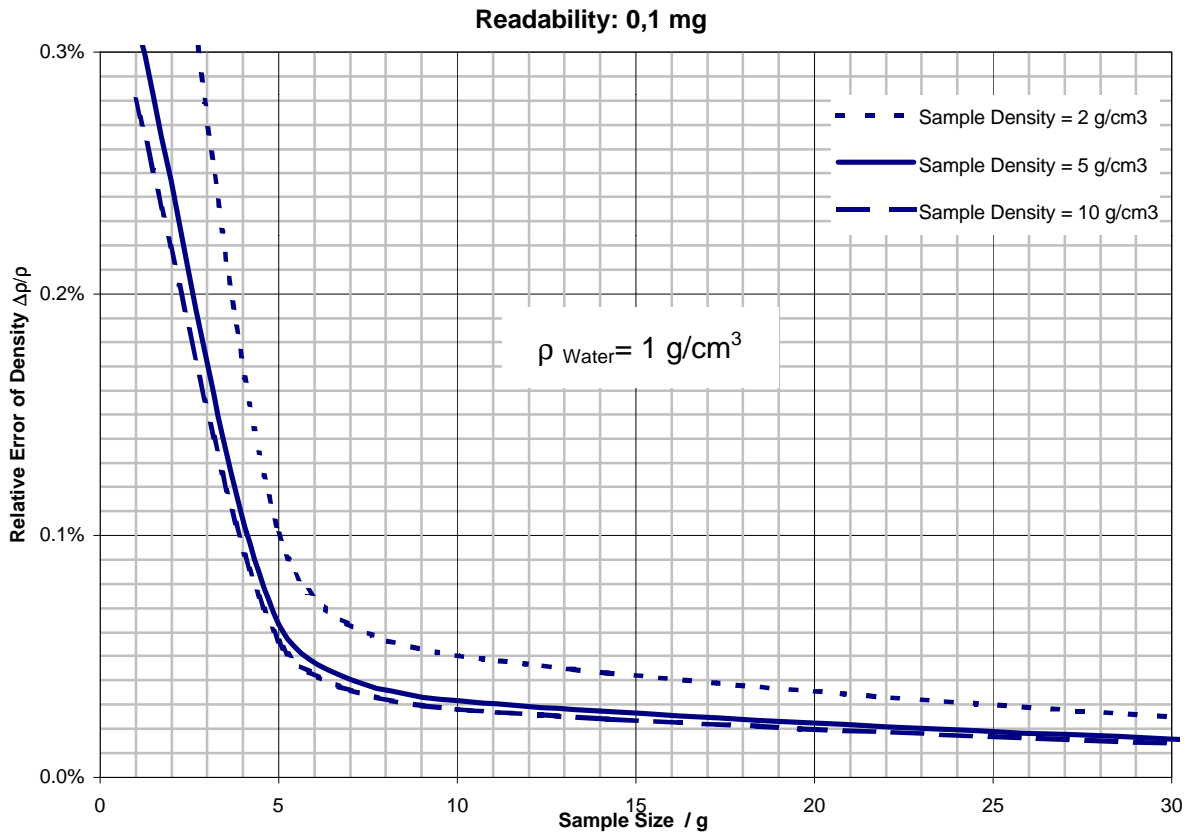


Figure 22: Solid density determination using the buoyancy method – the relative error of density is dependent on the sample size and the sample density

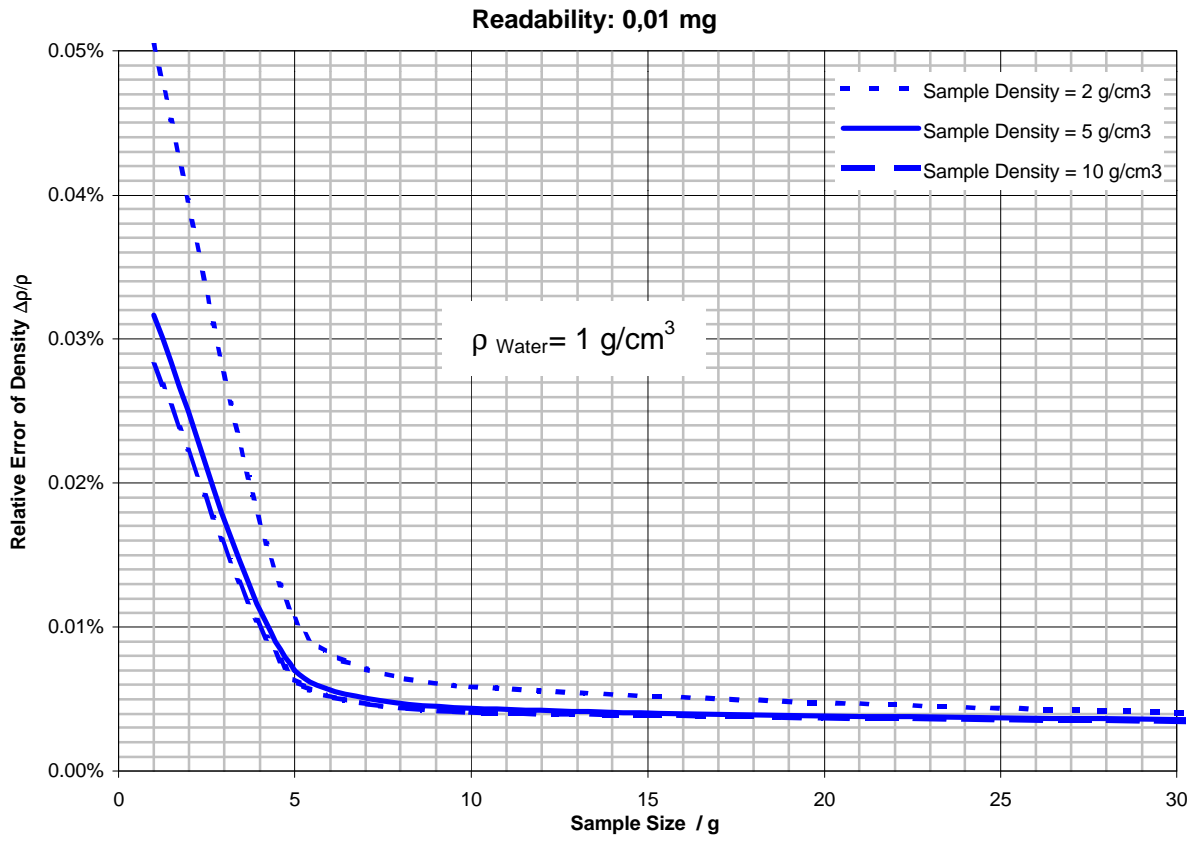


Figure 23: Solid density determination using the buoyancy method – the relative error of density is dependent on the sample size and the sample density

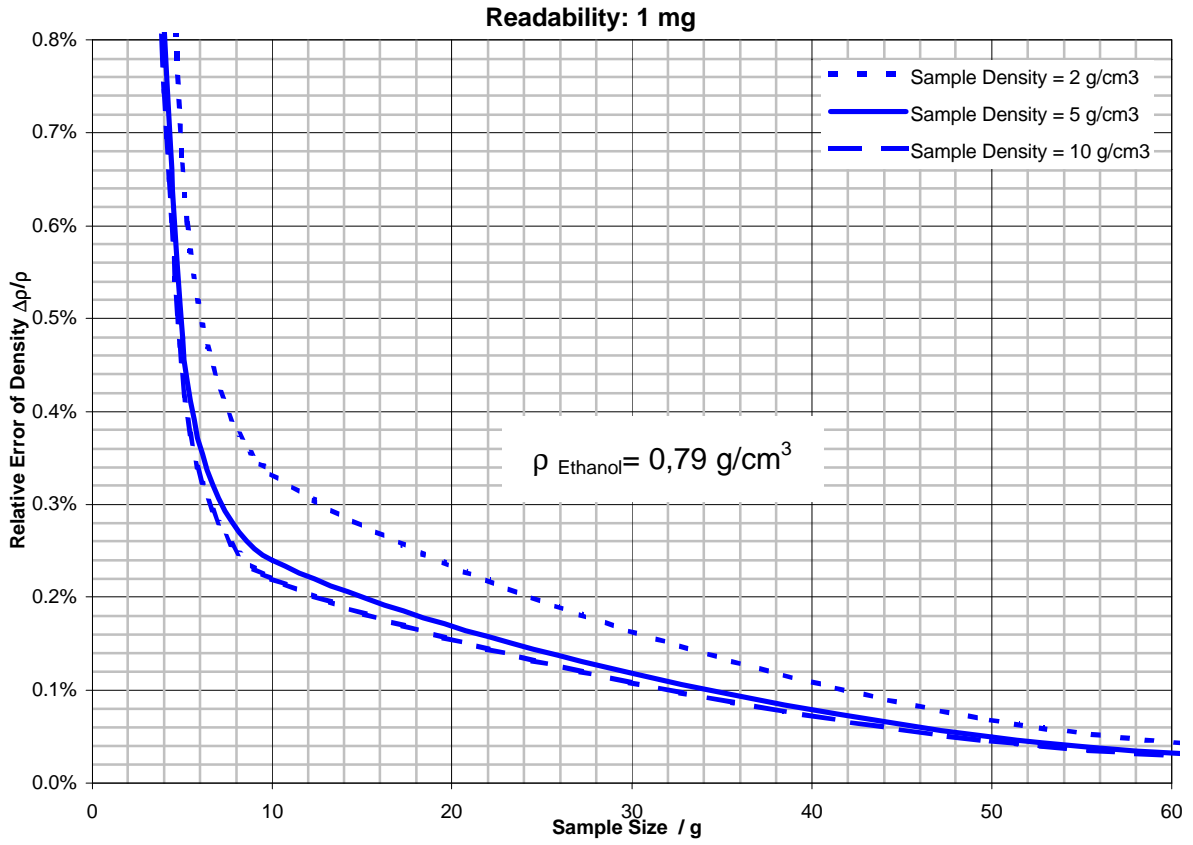


Figure 24: Solid density determination using the buoyancy method – the relative error of density is dependent on the sample size and the sample density

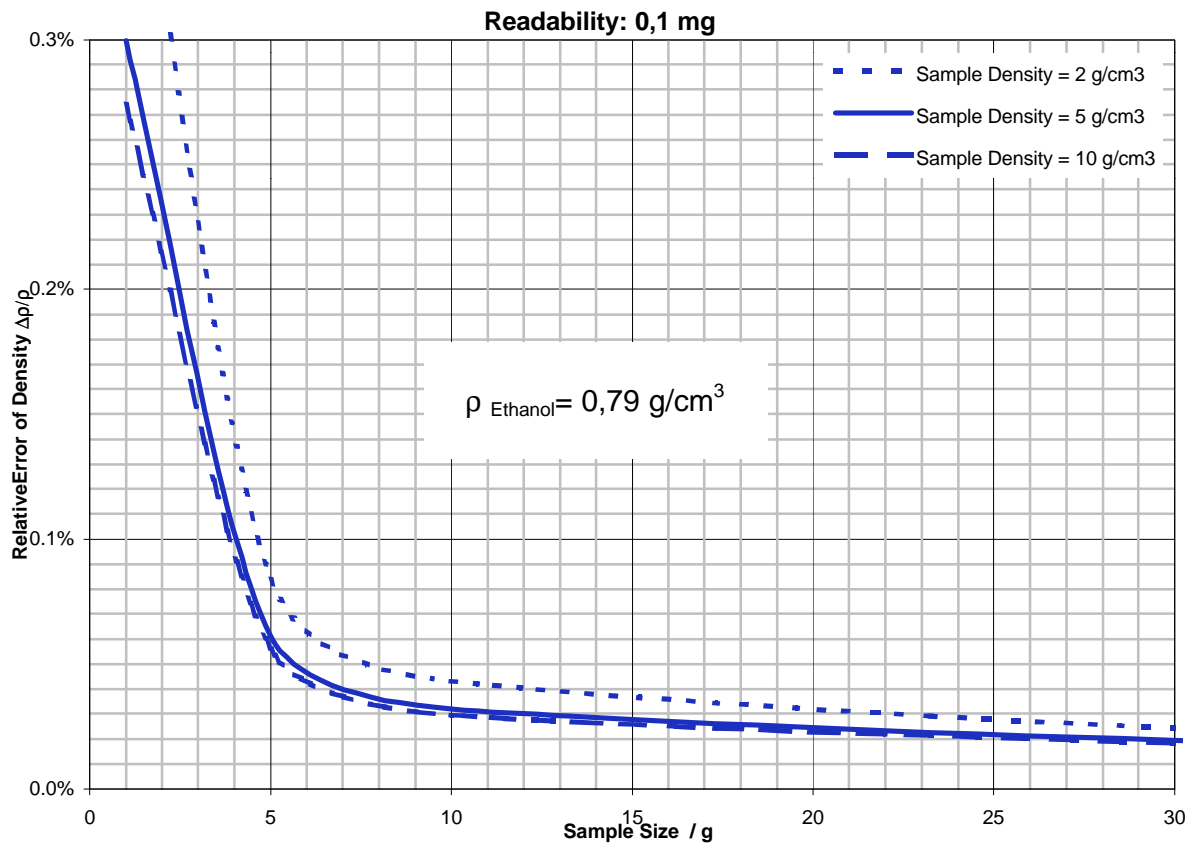


Figure 25: Solid density determination using the buoyancy method – the relative error of density is dependent on the sample size and the sample density

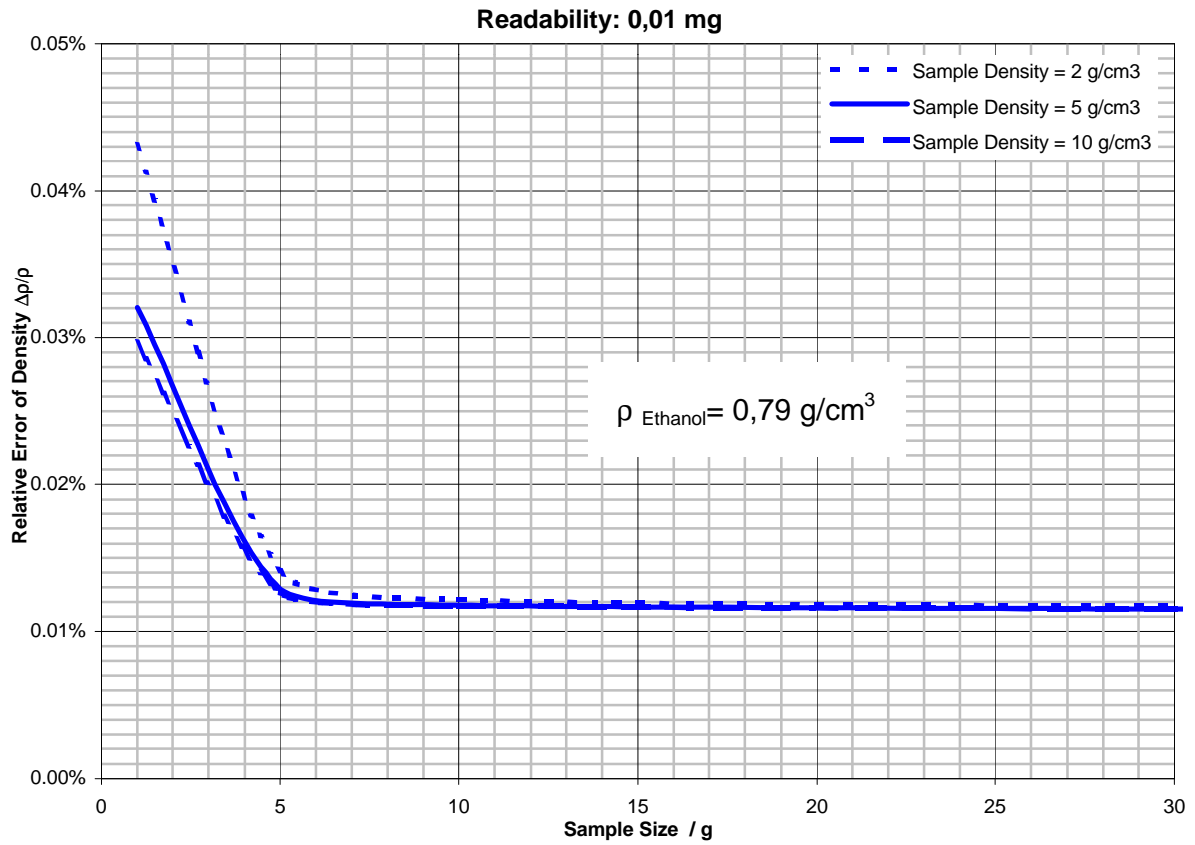


Figure 26: Solid density determination using the buoyancy method – the relative error of density is dependent on the sample size and the sample density

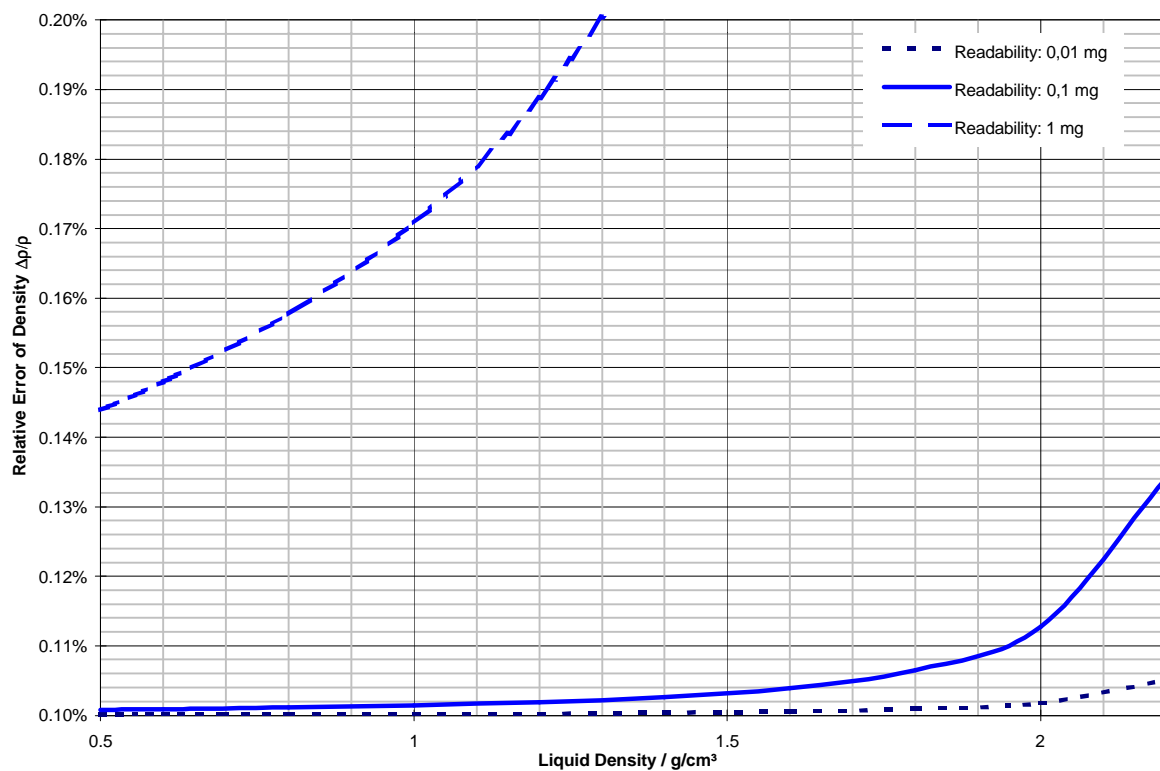


Figure 27: Liquid density determination using the buoyancy method – the relative error of density is dependent on the sample density and the readability of the weighing instrument for density determination with the sartorius Density Determination Set

Displacement Method

For solid body density determination using the displacement method the following relationship is applied, in which air density ρ_a is taken as constant and not used in the error calculation.

$$r_s = r_{fl} \cdot \frac{m_s}{m_{fl}} = (r_{fl} - r_a) \cdot \frac{W_s}{W_{fl}} + r_a$$

$$\frac{\Delta r}{r} = \sqrt{\left[\frac{\Delta r_{fl}}{r_{fl}} \right]^2 + \left[\frac{\Delta m_{(a)}}{m_{(a)}} \right]^2 + \left[\frac{\Delta m_{(fl)}}{m_{(fl)}} \right]^2}$$

In comparison with the buoyancy method, it is striking (see Figure 21 through 26) that solid body density determination using the buoyancy method results in a smaller error than the displacement method (see Figure 28) Aside from this it is clear that using the buoyancy method the mistake decreases with increasing sample density, whereas using the displacement method it increases with increasing sample density.

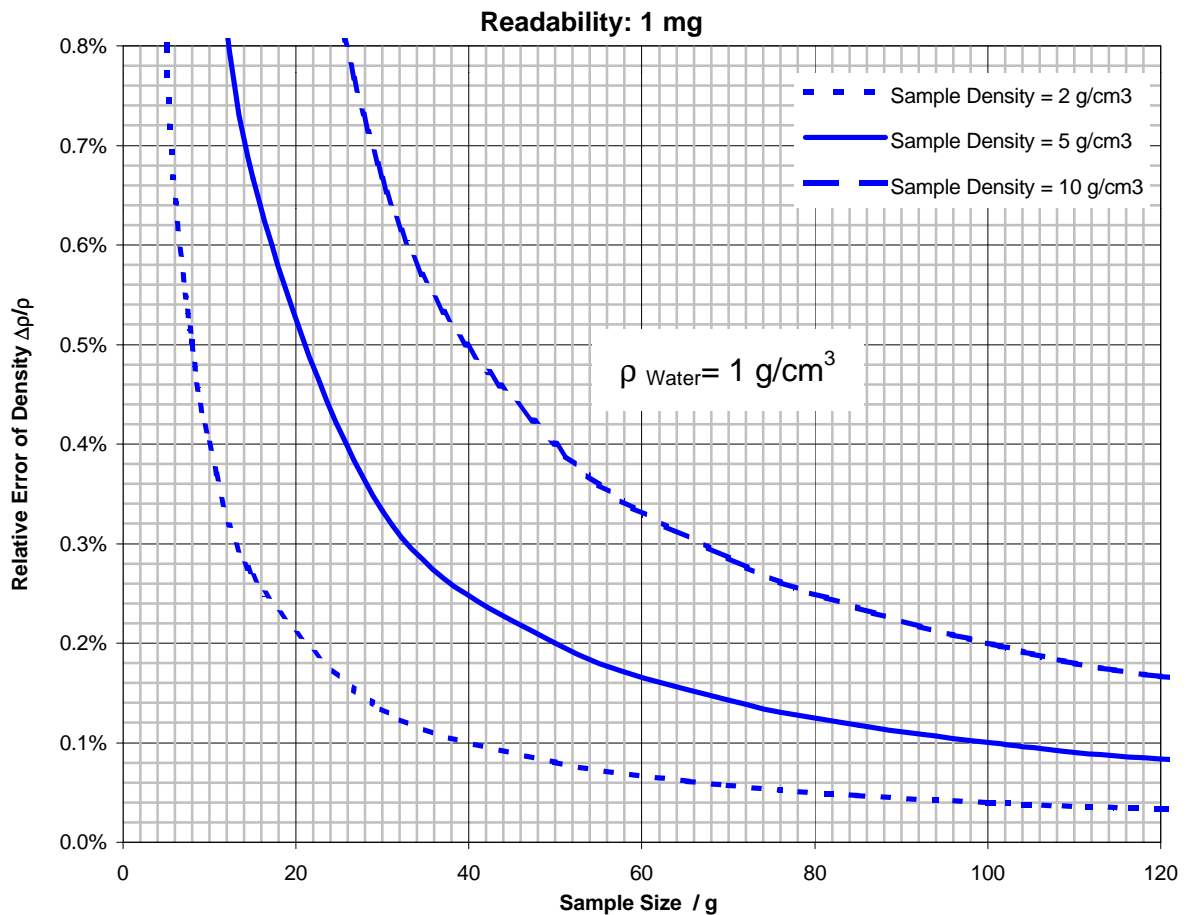


Figure 28: Solid density determination using the displacement method – the relative error of density is dependent on the sample size and the sample density

⁵ With $m_{(fl)} = \frac{r_{fl}}{r_s} \cdot m_{(a)}$

Pycnometer Method

For density determination using the pycnometer method, apply the relationship

$$\mathbf{r}_s = \mathbf{r}_{fl} \cdot \frac{m_2}{m_1 + m_2 - m_3} \quad \text{and} \quad \mathbf{r}_s = (\mathbf{r}_{fl} - \mathbf{r}_a) \cdot \frac{W_2}{W_1 + W_2 - W_3} + \mathbf{r}_a.$$

The error of air density is neglected again. To get the total error of the weight value, a linear error of one digit was added to the reproducibility of the weighing instrument type. For the error of liquid density a value of 0.00003 g/cm^3 is used, which corresponds to the value of a false readout of the thermometer of $\pm 0.1^\circ\text{C}$ – or a temperature change of $\pm 0.1^\circ\text{C}$. The liquid density with 1.0 g/cm^3 for water as medium is used in the equation below..

Next the error of the denominator $\Delta[m_1 + m_2 - m_3]$ is calculated

$$\Delta[m_1 + m_2 - m_3] = \sqrt{\Delta m_1^2 + \Delta m_2^2 + \Delta m_3^2}$$

and then the total relative error of density $\Delta\rho/\rho$

$$\frac{\Delta \mathbf{r}}{\mathbf{r}} = \sqrt{\left[\frac{\Delta \mathbf{r}_{fl}}{\mathbf{r}_{fl}} \right]^2 + \left[\frac{\Delta m_2}{m_2} \right]^2 + \left[\frac{\Delta(m_1 + m_2 - m_3)}{(m_1 + m_2 - m_3)} \right]^2}.$$

The following diagram (see Figure 29 to 31) for weighing instruments with different readabilities and weighing capacities shows the relative error of density determination dependent on the sample size and the sample density.

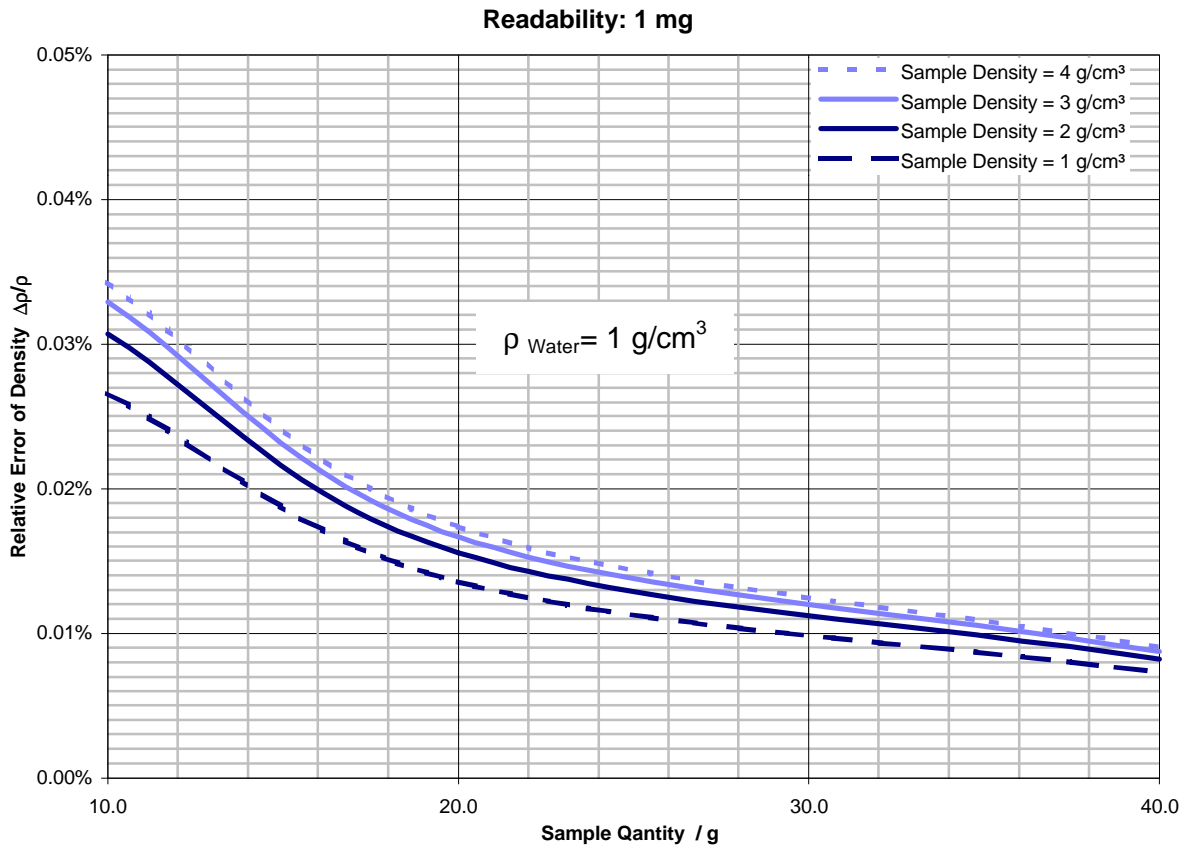


Figure 29: Density determination using the pycnometer method – the relative error of density is dependent on the sample size and the sample density.

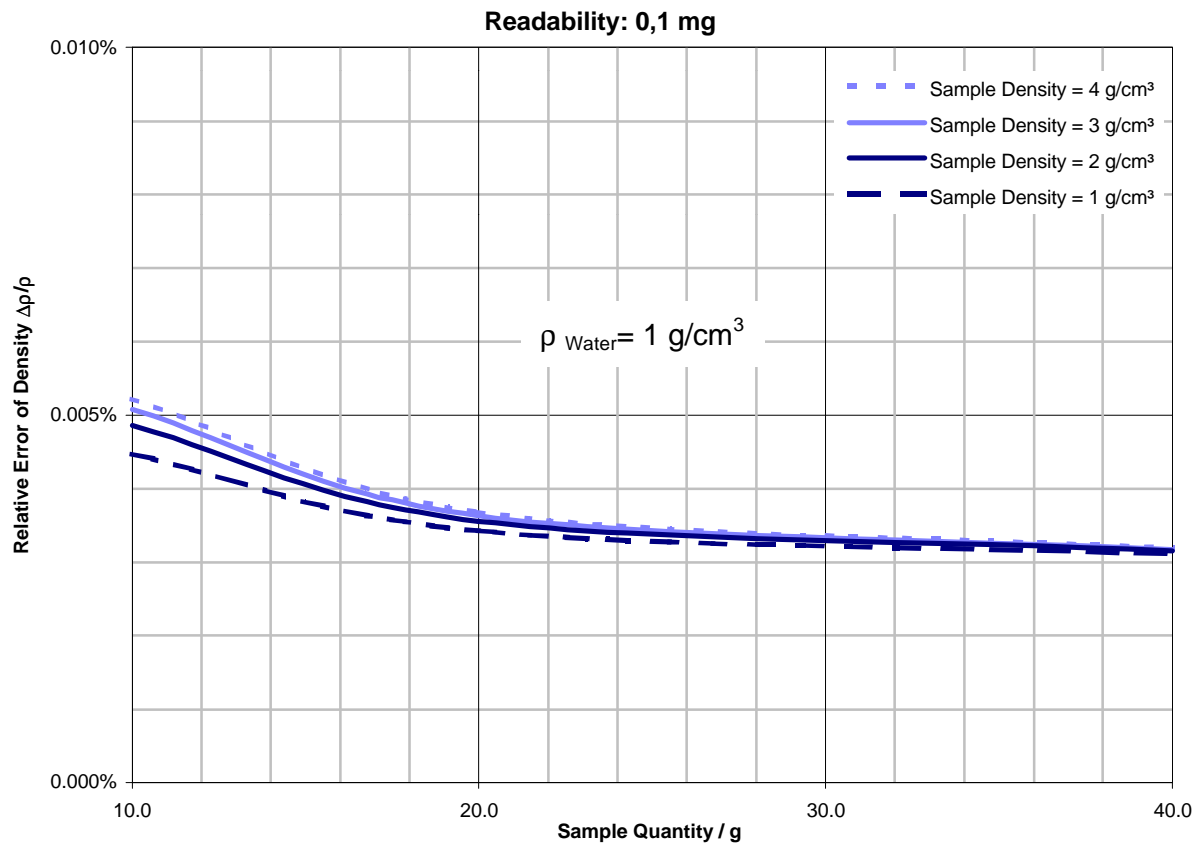


Figure 30: Density determination using the pycnometer method – the relative error of density is dependent on the sample size and the sample density.

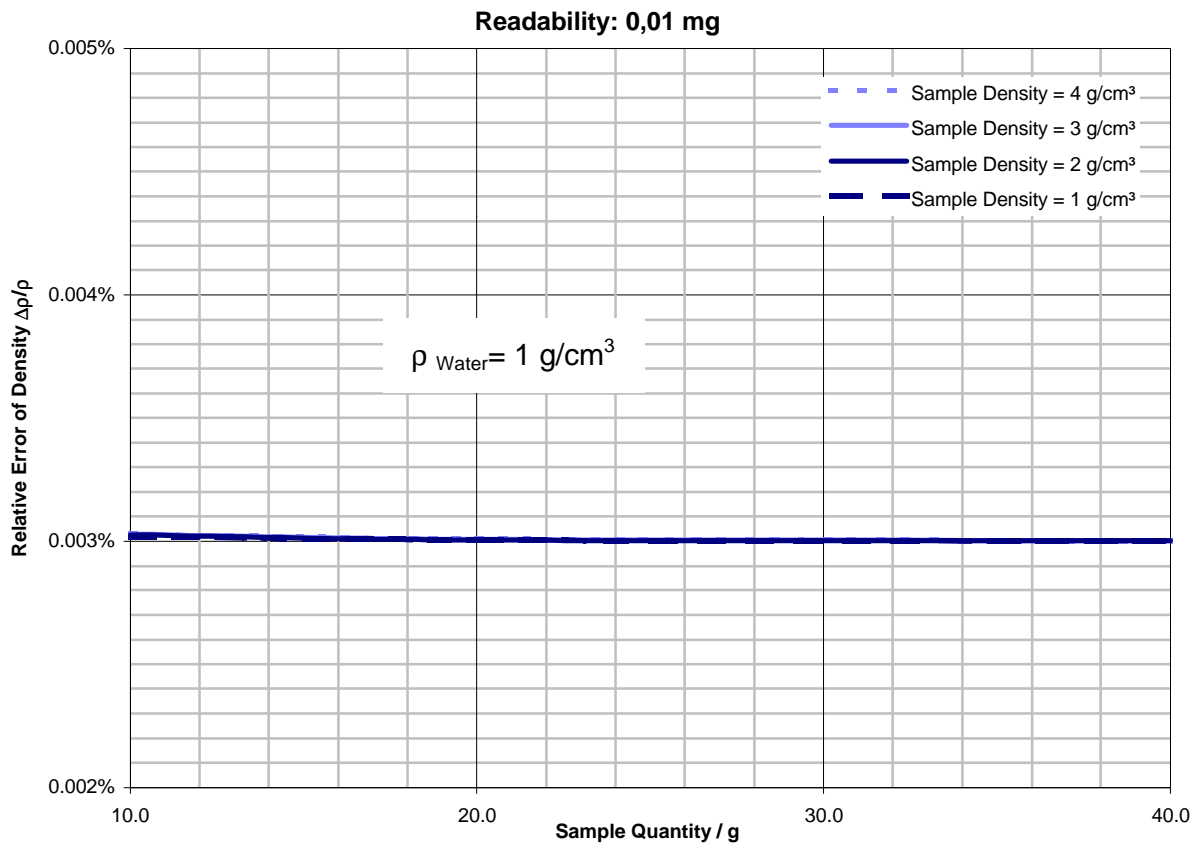


Figure 31: Density determination using the pycnometer method – the relative error of density is dependent on the sample size and the sample density.

Comparison of Different Methods of Density Determination

On the following pages different methods of density determination are contrasted, so that the most important advantages and disadvantages may be recognized at a glance.

	Buoyancy method	Displacement method	Pycnometer method	Weighing a defined volume
Suitable for:	Solids Liquids Dispersions Gases	Solids Liquids Dispersions	Solids Powder, Granules Liquids Dispersions	Powder (tap density) Liquids Dispersions
Advantages	Suitable for almost all sample types	Suitable for almost all sample types	Suitable for all sample types	
	Flexible with regard to sample size	Flexible with regard to sample size		
	Weighing instruments already available	Weighing instruments already available	Weighing instruments already available	Weighing instruments already available
	Quick process	Quick process	Accurate method	Quick process
	All methods especially easy to perform using weighing instruments with integrated software with user guidance prompts and evaluation of results			
Disadvantages	Solids and liquids must be brought to a defined temperature		Solids and liquids must be brought to a defined temperature	
	Large volume sample required for fluid density determination		Labor-intensive	
		Care must be taken to avoid evaporation	time-consuming	
	Sample must be wet very carefully	Sample must be wet very carefully		
	Bubbles must not be trapped	Bubbles must not be trapped	Bubbles must not be trapped	

	Buoyancy method	Displacement method	Pycnometer method	Weighing a defined volume
Uncertainty of measurement *	Dependent on the weighing instrument used and the sample amount and sample density		Dependent on the weighing instrument used	Dependent on the weighing instrument used
Readability 1 mg	Solids: <0,4% for m>10g	Solids: <0,2% for m>50g (water), $\rho=5\text{g/cm}^3$	<0,02% for m>20g	Liquids, Dispersions <1% for V=100ml, <0,1% for V=1000ml and $\rho<2\text{g/cm}^3$
	Liquids: ** <0,20% for $\rho=1,3\text{g/cm}^3$			
Readability 0,1 mg	Solids: <0,1% for m>5g	Solids: <0,02% for m>50g (water), $\rho=5\text{g/cm}^3$	<0,005% for m>10g	
	Liquids: ** <0,11% for $\rho<1,8\text{g/cm}^3$			
Readability 0,01 mg	Solids: <0,10% for m > 5g (water) <0,15% for m > 5g (ethanol)		<0,003% for m>10g	
	Liquids: ** -0,1% for $\rho<1,5\text{g/cm}^3$			

*For more exact results see the chapter entitled "Errors in and Precision of Density Determination"

**With the plummet from the Sartorius Density Determination Set

	Hydrometer	Oscillation-method	Density gradient column
Suitable for:	Liquids (Dispersions)	(Homogeneous) liquids	Solids (small sample size) (Liquids)
Advantages	Easy measurement	Small sample size: approx. 1ml	Several samples can be checked simultaneously
	Quick process	Quick process	
	Inexpensive plummet		
Disadvantages	In dispersions: measuring error can be caused by separation of the components in the sample		Time-consuming experiment preparation
		Density is slightly influenced by the viscosity of the sample	
		Expensive equipment must be purchased	
Uncertainty of measurement	0.1 to 10 kg/ m ³ or 0.0001 to 0.01 g/cm ³ for r=0.6 to 2.0 g/cm ³	There are measuring devices with uncertainty of measurement rated to 0.001g/cm ³ , 0.0001g/cm ³ or 0.00001g/cm ³	Calibrated glass references with densities of 0.8 to 2.0g/cm ³ ± 0.0002g/cm ³
	Hydrometers are suited to certain areas of sample surface tension		

Appendix

Temperature Dependency of Density

The dependence of density on temperature may be calculated with the aid of the volume expansion coefficient γ . In general, the expansion coefficient is only given for a set temperature range (e.g. 20 °C to 100 °C) for which a linear approximation is permissible. Numerical values for γ of different gases and liquids may be found in physical chemistry tables.

Usually, with solid bodies the expansion coefficient α is given. To convert the linear calculation into the volume expansion coefficient, use the relationship $\gamma \approx 3 \alpha$.

The density of a substance at temperature T_2 can be calculated with the aid of the temperature T_1 and the volume expansion coefficient:

$$\rho(T_2) = \frac{\rho(T_1)}{1 + \gamma(T_2 - T_1)}.$$

Hydrostatic Density Determination – Elimination of the Volumes in the Equation for ρ

With complete submersion of the body in the liquid, the experimental method results in the volume of the solid and the volume of the liquid being equal.

One can derive a relationship between the masses and densities of the two substances, in which volume is no longer explicitly included.

$$\mathbf{r}_{fl} = \frac{m_{fl}}{V_{fl}} \qquad \mathbf{r}_s = \frac{m_s}{V_s}$$

otherwise:

$$V_{fl} = \frac{m_{fl}}{\mathbf{r}_{fl}} \qquad V_s = \frac{m_s}{\mathbf{r}_s}$$

from $V_{fl} = V_s$

$$\frac{m_{fl}}{\mathbf{r}_{fl}} = \frac{m_s}{\mathbf{r}_s}$$

For determination of the solid density, it follows:

$$\mathbf{r}_s = \mathbf{r}_{fl} \cdot \frac{m_s}{m_{fl}}$$

Or for determination of the liquid density:

$$\mathbf{r}_{fl} = \mathbf{r}_s \cdot \frac{m_{fl}}{m_s} = \frac{m_{fl}}{V_s}$$

Air Density Determination

The relationship for determining air density is derived from the equations for the relationship between the mass and the weight value for aluminum and for steel:

$$W_{Al} = m_{Al} \cdot \frac{1 - \frac{r_a}{r_{Al}}}{1 - \frac{r_a}{r_N}} \qquad 1 = \frac{m_{Al}}{W_{Al}} \cdot \frac{1 - \frac{r_a}{r_{Al}}}{1 - \frac{r_a}{r_N}}$$

$$W_{St} = m_{St} \cdot \frac{1 - \frac{r_a}{r_{St}}}{1 - \frac{r_a}{r_N}} \qquad 1 = \frac{m_{St}}{W_{St}} \cdot \frac{1 - \frac{r_a}{r_{St}}}{1 - \frac{r_a}{r_N}}$$

Standardizing the equations to 1 and joining them gives:

$$\frac{m_{St}}{W_{St}} \cdot \frac{1 - \frac{r_a}{r_{St}}}{1 - \frac{r_a}{r_N}} = \frac{m_{Al}}{W_{Al}} \cdot \frac{1 - \frac{r_a}{r_{Al}}}{1 - \frac{r_a}{r_N}}$$

$$\frac{m_{St}}{W_{St}} \cdot \left(1 - \frac{r_a}{r_{St}}\right) = \frac{m_{Al}}{W_{Al}} \cdot \left(1 - \frac{r_a}{r_{Al}}\right)$$

$$W_{Al} \cdot m_{St} \cdot \left(1 - \frac{r_a}{r_{St}}\right) = W_{St} \cdot m_{Al} \cdot \left(1 - \frac{r_a}{r_{Al}}\right)$$

$$W_{Al} \cdot m_{St} - W_{Al} \cdot m_{St} \cdot \frac{r_a}{r_{St}} = W_{St} \cdot m_{Al} - W_{St} \cdot m_{Al} \cdot \frac{r_a}{r_{Al}}$$

$$W_{St} \cdot m_{Al} \cdot \frac{r_a}{r_{Al}} - W_{Al} \cdot m_{St} \cdot \frac{r_a}{r_{St}} = W_{St} \cdot m_{Al} - W_{Al} \cdot m_{St}$$

$$W_{St} \cdot m_{Al} - W_{Al} \cdot m_{St} = r_a \cdot \left(\frac{W_{St} \cdot m_{Al}}{r_{Al}} - \frac{W_{Al} \cdot m_{St}}{r_{St}} \right)$$

$$r_a = \frac{W_{St} \cdot m_{Al} - W_{Al} \cdot m_{St}}{\left(\frac{W_{St} \cdot m_{Al}}{r_{Al}} - \frac{W_{Al} \cdot m_{St}}{r_{St}} \right)}$$

Air Buoyancy Correction

With the example of density determination by the displacement method, the formula for calculating the density with regard to air buoyancy is derived:

Density is calculated by $\rho_s = \rho_{fl} \cdot \frac{m_s}{m_{fl}}$,

for the mass the following relationship that describes the dependency of the mass on air density is

substituted: $m = W_v \cdot \frac{1 - \frac{\rho_a}{\rho_G}}{1 - \frac{\rho_a}{\rho}}$

$$\rho_s = \rho_{fl} \cdot \frac{W_s \cdot (1 - \frac{\rho_a}{\rho_G}) \cdot (1 - \frac{\rho_a}{\rho_{fl}})}{(1 - \frac{\rho_a}{\rho_s}) \cdot W_{fl} \cdot (1 - \frac{\rho_a}{\rho_G})}$$

$$\rho_s = \rho_{fl} \cdot \frac{W_s \cdot (1 - \frac{\rho_a}{\rho_{fl}})}{(1 - \frac{\rho_a}{\rho_s}) \cdot W_{fl}}$$

$$\rho_s = \frac{W_s}{W_{fl}} \cdot \frac{\rho_{fl} - \rho_a}{1 - \frac{\rho_a}{\rho_s}}$$

$$\rho_s = \frac{W_s}{W_{fl}} \cdot (\rho_{fl} - \rho_a) \cdot \frac{1}{\frac{\rho_s}{\rho_s} - \frac{\rho_a}{\rho_s}}$$

$$\rho_s = \frac{W_s}{W_{fl}} \cdot (\rho_{fl} - \rho_a) \cdot \frac{1}{\frac{(\rho_s - \rho_a)}{\rho_s}}$$

$$\rho_s = \frac{W_s}{W_{fl}} \cdot (\rho_{fl} - \rho_a) \cdot \frac{\rho_s}{\rho_s - \rho_a}$$

$$\rho \cdot \frac{\rho_s - \rho_a}{\rho_s} = \frac{W_s}{W_{fl}} \cdot (\rho_{fl} - \rho_a)$$

$$\rho_s - \rho_a = \frac{W_s}{W_{fl}} \cdot (\rho_{fl} - \rho_a)$$

$$\rho_s = \frac{W_s}{W_{fl}} \cdot (\rho_{fl} - \rho_a) + \rho_a$$

Questions About Density

1. How is density defined and what is the unit of density?
2. How does density change when temperature increases?
3. Why do bodies appear to be lighter in water than in air when they are weighed? Describe the phenomenon.
4. When do bodies "float" in a liquid? Which statement then is valid for the density of liquids and solids?
5. What is the difference between the structure of the experiment for density determination using the buoyancy and using the displacement method, and what do the measured values include with each method?
6. With which method can the density of fluids be determined? – Why are fluid densities determined; which conclusions can one reach from the measured values?
7. What is a dispersion and how can one measure its density?
8. What is the difference between density and bulk density – how can one determine the density of porous material?
9. What is the density of air; when must one know the air density and how can one determine it?
10. How do you determine the density of powders?
11. Which product or material properties can be controlled by measuring density?
12. What relevance has density determination in the realm of the prepackage industry for average weight control?
13. On what is the accuracy of density determination dependent and how does one measure the error of density values?
14. Explain the meaning of the individual symbols and factors in the following formulas; what is included in Corr?
$$\text{Rho} = (\text{Wa} * (\text{Rhofl} - \text{LA})) / ((\text{Wa} - \text{Wfl}) * \text{Corr}) + \text{LA}$$
$$\text{Rho} = (\text{Wa} * (\text{Rhofl} - \text{LA})) / (\text{Wfl} * \text{Corr}) + \text{LA}$$
$$\text{Rho} = (\text{Wa} * (\text{Rhofl} - \text{LA})) / (\text{Wfl} + \text{Wa} - \text{Wr}) + \text{LA}$$
15. Which advantages are offered by the application tare memory in the LA/FC weighing instruments in density determination with the pycnometer method?
16. On products such as mustard, the amount of the full package must be given in ml. During filling and checkweighing, however, the weight (or mass) is checked. The density is necessary as a conversion factor between mass and volume. Which density determination method do you recommend to your customer
 - when high accuracy is desired?

- when a quick result is important?
17. A client wants to know the density of melted glass (in a laboratory oven with the opening on top) at 1200°C – what do you recommend?

Tips for answering the questions

1. Density = Mass / Volume
 $1000 \text{ kg/m}^3 = 1 \text{ kg/dm}^3 = 1 \text{ g/cm}^3 = 1 \text{ g/ml}$
2. The density decreases (see p. 3).
3. Resultant force = Weight force minus buoyancy force.
The buoyancy is dependent on the hydrostatic pressure in the liquid $p = \rho \cdot g \cdot h$;
 $F = p \cdot A = \rho \cdot g \cdot h \cdot A = \rho \cdot g \cdot V$ (see p. 7 – 8)
4. The density of the solid body and the liquid are the same (see p. 9).
5. The beaker with liquid for buoyancy stands on the weighing pan using the displacement method, it has no contact with the weighing pan using the buoyancy method.
Measured value corresponds with the mass of the displaced fluid using the displacement method.
Measured value corresponds with the mass reduced by the buoyancy in the buoyancy method (see p. 10).
6. Buoyancy method, displacement method, weighing a defined volume, hydrometer, oscillation method, suspension method (not with mixable fluids).
Conclusion from the concentration relationships, AWC: gravimetric rather than volumetric filling; see p. 27 and table, p. 50 - 52
7. Multi-phase system consisting of continuous phase (matrix) and one or several finely divided phases (dispersed phases) (see p. 28)
Choice of the method is dependent on the accuracy required, the consistency and flowing property of the dispersion: either as for liquids or the pycnometer method. Be careful of the influence of phase separation using the different methods, with the pycnometer method the separation has no effect on the result.
8. With porous material: bulk density relates to the total volume including open and closed pores – true density relates to the solid volume.
Density determination methods with porous samples: Buoyancy method (see p. 22) or pycnometer method for determination of true density after grinding the sample to a powder. Advantage of the pycnometer method: a higher measured accuracy is possible, existing closed pores are not attributed to the solid.
9. $\rho_{\text{Luft}} = 0,0012 \text{ g/cm}^3$
With analytical and microbalances the weighed value must be corrected to yield the mass accurately.
With two weights of different densities and thereby different volumes, and of more or less the same mass, air density can be determined with a microbalance.
The conventional mass value of the weights and the density of the material must be known before air

density can be calculated from the measured values (see p. 13 – included in the software of Sartorius micro- and ultra-microbalances).

10. Using the pycnometer method. For a description, see p. 16, 25.
11. Porosity, voids, crystal content (crystalline phases have a higher density than non-crystalline glass phases), cooling rate with glass, concentration of an ingredient in a solution, solid component in suspensions ... see p. 3.
12. Volumetry is replaced by gravimetry, a more accurate and simpler measuring method. The proportionality factor of mass and volume is the density (verified devices must be used).
13. Careful maintenance of the experimental conditions, for example temperature ... see p. 35. The error of the measured value is dependent on readability, repeatability ... of the weighing instrument. The total error of the result must be calculated using the rules of error reproduction see p. 36.
14. See instructions for the LA and FC weighing instrument models.
15. Save time because the procedure for drying the pycnometer can be skipped; see p. 26.
16. ???
17. ???

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